



**TECHNICAL UNIVERSITY OF MOMBASA**  

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**FACULTY OF APPLIED AND HEALTH SCIENCES**  
**DEPARTMENT OF MATHEMATICS & PHYSICS**  
**UNIVERSITY EXAMINATION FOR:**  
**BMCS/BSSC**  
**AMA 4215: STATISTICS III**  
**END OF SEMESTER EXAMINATION**  
**SERIES: APRIL 2016**  
**TIME: 2 HOURS**  
**DATE: 20 May 2016**

**Instructions to Candidates**

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of Choose No questions. Attempt Choose instruction.

**Do not write on the question paper.**

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**Question ONE (30 Marks)**

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- (a) Fatal accidents occur at random at a known 'black spot', following a Poisson process with mean 4 per year.
- (i) Draw a diagram of the probability mass function of X, the actual annual number of fatal accidents. (4 marks)
- (ii) Determine the probability that in a given year there is at most one fatal accident. (2 marks)
- (b) The random variable X follows the exponential distribution with rate parameter  $\lambda$ , so that the probability density function (pdf) of X is given by

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0, \lambda > 0.$$

Given that the moment generating function of X,  $M_X(t)$  say, is given by

$$M_X(t) = \left(1 - \frac{t}{\lambda}\right)^{-1}, \quad t < \lambda,$$

show that the mean and variance of X are given by  $\frac{1}{\lambda}$  and  $\frac{1}{\lambda^2}$  respectively (6 marks)

- (c) The bivariate probability distribution of the random variables  $X$  and  $Y$  is summarised in the following table.

		Y			
		0	1	2	3
X	0	$k$	$6k$	$9k$	$4k$
	1	$8k$	$18k$	$12k$	$2k$
	2	$k$	$6k$	$9k$	$4k$

- (i) Find  $k$ . (3 marks)  
 (ii) Obtain the marginal distributions of  $X$  and  $Y$ . (4 marks)
- (d) The distribution  $f_{X,Y}(x, y) = \frac{1}{\sqrt{(2\pi)}} e^{-0.5(x^2+y^2)}$  is a bivariate normal distribution. For this distribution, state  $\sigma_X, \sigma_Y, \mu_X, \mu_Y, \rho$  (4 marks)
- (e) The joint density function of 2 random variables  $X$  and  $Y$  is given as

$$f_{X,Y}(x, y) = \begin{cases} \frac{2}{5}(2x + 3y), & 0 < x < 1, \quad 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

- (i) Verify that  $f_{X,Y}(x, y)$  is a pdf (4 marks)  
 (ii) Find  $P[(X, Y)] \in A$ , where  $A = \{(x, y) \mid 0 < x < \frac{1}{2}, \frac{1}{4} < y < \frac{1}{2}\}$  (3 marks)

### Question TWO (20 Marks)

- (a) Suppose  $X$  and  $Y$  are independent random variables having Poisson distributions with respective means  $\lambda (> 0)$  and  $\mu (> 0)$ .
- (i) Show that  $X + Y$  also follows a Poisson distribution. (5 marks)  
 (ii) Find  $P(X = k \mid X + Y = n)$  (when  $k$  and  $n$  are integers with  $0 \leq k \leq n$ . For given fixed  $n > 0$ , name the distribution you have obtained. (7 marks)
- (b) Telephone calls arriving at a computer helpline are classed as urgent or standard; urgent calls average 8 per hour, standard calls average 24 per hour. Ten calls arrive within 30 minutes; find (to two significant figures) the probability that at most two of them are urgent, stating any assumptions you make. (8 marks)

### Question THREE (20 Marks)

The continuous random variables  $X$  and  $Y$  have joint probability density function  $f(x, y) = kxy$  if  $0 < x < y < 1$ , with  $f(x, y) = 0$ , elsewhere, where  $k$  is a constant.

- (a) Evaluate  $k$ , and find the marginal probability densities of  $X$  and  $Y$ . Say, with a reason, whether or not  $X$  and  $Y$  are independent. (10marks)  
 (b) Show that, for all non-negative integers  $r$  and  $s$ ,  $E(X^r Y^s) = \frac{8}{(r+2)(r+s+4)}$   
 Hence find the correlation between  $X$  and  $Y$ . (10marks)

### Question FOUR(20 Marks)

- (a) Suppose  $X$  and  $Y$  are independent random variables, each following the chis-squared distribution with four degrees of freedom; this distribution has probability density function (pdf)  $w e^{-w/2} / 4$  on  $w > 0$ .

Define new random variables  $U = X/Y$  and  $V = Y$ . Obtain the joint pdf of  $U$  and  $V$ , and hence show that  $U$  has pdf  $h(u) = (6u)/(1+u)^4$  for  $u > .0$  (10marks)

[You may use without proof the result  $\int_0^\infty t^k e^{-t} dt = k!$  when  $k$  is a positive integer.]

Name the distribution followed by  $U$  in (a). Using the density function, show that  $E(U) = 2$ , and hence verify that  $E(X/Y) > E(X) / E(Y)$ . Explain briefly why this is no surprise. (10marks)

**Question FIVE(20 Marks)**

(a) The random variables  $X_1, X_2$  have the bivariate Normal distribution with expectation  $u = (\mu_1 \ \mu_2)^T$  covariance

$$\text{matrix } \Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

- (i) Write out explicitly the joint probability density function of  $X_1$  and  $X_2$ . (3 marks)
- (ii) State (without proof) the marginal distribution of  $X_2$  and write out its marginal probability density function. (1mark)
- (iii) Hence obtain the conditional probability density function of  $X_1$  given that  $X_2 = x_2$ . Identify this as a Normal distribution with parameters that you should state explicitly. (6 marks)

(b) The random variables  $X_1, X_2, X_3$  have the multivariate Normal distribution with expectation  $u =$

$$(\mu_1 \ \mu_2 \ \mu_3)^T \text{ and covariance matrix, } \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix}. \text{ Let } \Sigma_{23} = \begin{pmatrix} \sigma_2^2 & \sigma_{23} \\ \sigma_{23} & \sigma_3^2 \end{pmatrix}, \text{ a sub-matrix of } \Sigma. \text{ In}$$

general, the conditional distribution of  $X_1$  given that  $X_2 = x_2, X_3 = x_3$  is a Normal distribution with

$$E(X_1 | x_2, x_3) = \mu_1 + (\sigma_{12} \ \sigma_{13}) \Sigma_{23}^{-1} \begin{pmatrix} x_2 - \mu_2 \\ x_3 - \mu_3 \end{pmatrix},$$

$$\text{Var}(X_1 | x_2, x_3) = \sigma_1^2 - (\sigma_{12} \ \sigma_{13}) \Sigma_{23}^{-1} \begin{pmatrix} \sigma_{12} \\ \sigma_{13} \end{pmatrix}.$$

Obtain the parameters of the conditional distribution of  $X_1$  given that  $X_2 = x_2, X_3 = x_3$  in the special case where  $X_2$  and  $X_3$  are independent random variables. Find an expression for the multiple correlation of  $X_1$  on both  $X_2$  and  $X_3$  in this case. (10 marks)