TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BMCS/BSSC

AMA 4215: STATISTICS III

END OF SEMESTER EXAMINATION

SERIES:APRIL2016

TIME:2HOURS

DATE:20May2016

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of Choose No questions. AttemptChoose instruction.

Do not write on the question paper.

Question ONE (30 MarkS)

(a) The table below shows the joint distribution of two random variables, X and Y.

		Values of Y			
		1	2	3	4
Values of X	1	$6c$	$3c$	$2c$	$4c$
	2	$4c$	$2c$	$4c$	0
	3	$2c$	c	0	$2c$

(i) Find c. (2 marks)

(ii) Calculate the marginal distributions of X and Y. (5 marks)

(b) A passenger aircraft carries 100 passengers. Assume that the weight in kg, W say, of a randomly chosen passenger is Normally distributed with mean 65 and standard deviation 6, i.e. $W \sim N(65, 36)$. Assume also that a randomly chosen passenger has hold luggage weighing H kg, where $H \sim N(20, 4)$, and cabin luggage weighing C kg, where $C \sim N(6, 9)$, W, H and C being independent within and between passengers.

(i) Write down the distribution of the total weight in kg, T say, of a randomly chosen passenger and his or her hold and cabin luggage, and find $P(T > 110)$ (4 marks)

(ii) A safety requirement is that the total weight of all 100 passengers and their luggage should not exceed 9300 kg. Find the probability that this requirement is not met. (3 marks)

(c) Each of the following problems could occur in fitting a multiple linear regression model. For each of the problems suggest a possible solution. Justify your answers.

(i) The residuals are uncorrelated but have a non-constant variance. (2 marks)

(ii) The overall model is statistically significant, but none of the individual parameter estimates is significant. (2 marks)

(d) State the Gauss-Markov theorem. (3 marks)

(e) The continuous random variables X and Y have joint probability density

Function $f(x, y) = 12x^2, \quad 0 \le x \le y \le 1$

Derive the marginal probability density functions of X and Y. Using the results (5marks)

(f) The continuous random variables X and Y independently follow the uniform distribution on the interval 0 to 1. The random variables U and V are defined by

$$U = (-lnX)^{1/2}\sin(2\pi Y), \qquad V = (-lnX)^{1/2}\cos(2\pi Y)$$

Show that $X = \exp\left[-\frac{1}{2}\left(U^2 + V^2\right)\right]$ and $Y = \frac{1}{2\pi}\tan^{-1}\left(\frac{U}{V}\right)$.

(4 marks)

Question TWO (20 MarkS)

Snapdragon (*Antirrhinum*) plants can have red, pink or white flowers; the flowers on any individual plant are all the same colour. A snapdragon plant grown from a dihybrid cross has probability ¼ of having red flowers, ½ of having pink flowers and ¼ of having white flowers. A gardener grows 20 snapdragon plants, each having been independently produced from a dihybrid cross. Let the random variable X be the number of these plants that have red flowers and Y the number of them that have pink flowers.

(a) Write down an expression for the joint probability distribution, $P(X = x, Y = y)$.

Find the probability that the gardener grows exactly five plants with red flowers *and* exactly ten plants with pink flowers. (5marks)

(b) The random variable X follows a binomial distribution. Without doing any algebra, explain why, and state its parameters. (4 marks)

(c) Find the probability that the gardener grows no more than one plant with white flowers. (4 marks)

(d) The conditional distribution of X, given $Y = y$ (for any possible value y), is also a binomial distribution. Without doing any algebra, explain why. State the parameters of this conditional distribution. (3marks)

(e) Assume that the time of flowering does not vary with colour, and that the first five plants to flower are all pink. Find the probability that at least three of the remaining plants will have red flowers. (4 marks)

Question THREE (20 MarkS)

(a) Suppose that X and Y are independent random variables, each following the chi-squared distribution with one degree of freedom. This distribution has probability density function (pdf)

$$f(w) = \frac{1}{\sqrt{2\pi w}} e^{-w/2}, \qquad w > 0.$$

Define new random variables U and V as follows:

$U = X/Y, \quad V = Y$

Obtain the joint pdf of U and V, and hence show that U has pdf

$$\frac{1}{\pi(u+1)\sqrt{u}}$$

for $u > 0$. (14 marks)

(b) Suppose now that W_1 and W_2 are independent $N(0, \sigma^2)$ random variables, for some $\sigma > 0$. Noting that (i) shows that U follows the $F_{1,1}$ distribution, or otherwise, find the pdf of $|W_1/W_2|$ (6 marks)

Question FOUR(20 Marks)

(a) The continuous random variable X follows the gamma distribution with probability density function

$$f_X(x) = \frac{\theta^\alpha x^{\alpha-1} e^{-\theta x}}{\Gamma(\alpha)}, \qquad x > 0.$$

Here α and θ are positive constants and Γ(.) denotes the gamma function,

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$$

Show that X has the following moment generating function (mgf):

$$M_X(t) = \left(\frac{1}{1-(t/\theta)}\right)^\alpha, \qquad t < \theta$$

Hence find the expected value and variance of X (9 marks)

(b) Using the result of part (a) or otherwise find the mgf of the standardized variable

$$Z = \frac{\theta}{\sqrt{\alpha}}\left(X - \frac{\alpha}{\theta} \right).$$

Now let θ be fixed. Find the limiting form of this mgf as $\alpha \to \infty$. (11marks)

Question FIVE(20 Marks)

A random sample of size n is drawn from the uniform distribution on the interval 0 to θ (where $\theta > 0$). The ordered values in this sample are $X_{(1)} \le X_{(2)} \le \dots \le X_{(n)}$. The sample range is $U = X_{(n)} - X_{(1)}$.

(a) (i) Write down the probability density function (pdf) of $X_{(j)}$, for $j = 1, 2, \dots, n$.

(ii) Hence obtain the expected value and variance of $X_{(j)}$.

(iii) Deduce the expected value of U. (9 marks)

(b) Find the joint pdf of $X_{(1)}$ and $X_{(n)}$. Use it to find $E(U^2)$ and deduce That the variance of U is

$$\frac{2(n-1)\theta^2}{(n+1)^2(n+2)}$$

(7 marks)

(c) The statistics

$$\frac{n+1}{n}X_{(n)} \text{ and } \frac{n+1}{n-1}U$$

both have expected value θ and are both possible estimators of θ. Find their variances. Which do you think is the better estimator? (4 marks)