



TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE DEGREE OF:

BACHELOR OF SCIENCE IN STATISTICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER,

BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY SCIENCE

SMA 2101/AMA 4103: CALCULUS 1

END OF SEMESTER EXAMINATION

TIME: 2 HOURS

SERIES: MAY 2016

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of 5 questions. **ATTEMPT QUESTION ONE AND ANY OTHER TWO.**

Do not write on the question paper.

Question ONE

- (a) Find the first four derivatives of the function $f = \sin(3y) + e^{-2y} + \ln(7y)$ (4mks)
- b) Determine the stationary points of $g(x) = 2x^3 - 9x^2 + 36$ hence sketch the graph (7mks)
- c) Given $f(x) = 3x^2 - x + 10$ and $g(x) = 1 - 20x$ find
- i) $(f \circ g)(x)$ (4mks)
- ii) $(g \circ f)(x)$ (2mks)

d) A rectangle is to be inscribed in a circle of radius 4cm, determine the largest possible area of the rectangle (5mks)

e) Evaluate the following limits

$$\lim_{x \rightarrow \infty} \frac{x+1}{2x+1} \quad (3\text{mks})$$

$$\lim_{x \rightarrow 0} x \cot x \quad (3\text{mks})$$

f) Find the derivative of $g(t) = \frac{t}{t+1}$ using first principal (5mks)

Question TWO

a) Find the maximum and minimum values of the function $f(x) = x^3 - 2x^2 - 5x + 6$ hence sketch the curve (10mks)

b) Evaluate the following limits:

i) $\lim_{x \rightarrow 5} \frac{x^2 - 18x + 15}{x - 5}$ (2mks)

ii) $\lim_{x \rightarrow 0} \frac{\cos x}{2}$ (2mks)

iii) $\lim_{x \rightarrow \infty} \frac{6x-1}{2x-1}$ (2mks)

c) Given $f(x) = 2x+1$ and $g(x) = \frac{x}{3}$. Find $(g \circ f)^{-1}$. (4mks)

Question 3 (20marks)

a) A rectangular plot is to be enclosed by 300m of fencing mesh what is the maximum possible area that can be enclosed (7mks)

b) State 3 conditions for a function $f(x)$ to be continuous at a point $x = a$ (3mks)

c) Find the derivatives of the following functions.

i) $y = \sqrt{x(x+1)}$ (2marks)

ii) $y = \frac{\sin x}{\ln x}, x > 0$ (4mks)

iii) $y = 2x^3 e^{3x}$ (4mks)

Question 4 (20marks)

a) Show that $\frac{dy}{dx} = \arcsin x = \frac{1}{1-x^2}$ (4mks)

b) Find the derivative of $xy + x - 2y = 5$ (4mks)

c) Evaluate the following limits

i) $\lim_{x \rightarrow 0} \frac{x}{\sin 7x}$ (4mks)

ii) $\lim_{x \rightarrow 0} \frac{\tan x}{x}$ (4mks)

iii) $\lim_{x \rightarrow \infty} \frac{4x^3 - 21x + 6}{3x^3 + x^2 - 9}$ (4mks)

Question 5 (20 marks)

a) $f(x) = x^4 + 2x^3 - 3x^2 - 4x + 4$ find the critical points of the function. Hence sketch the curve (10mks)

b) An open cylindrical container with circular base is to hold 64 litres of water. Find its dimensions so that the amount of metal required is minimum (4mks)

c) Find the tangent and normal to the function $y = \frac{3x^2 - 1}{x}$ at the point (1,2) (3mks)

d) A particle rotates anticlockwise from rest according to $\theta = \frac{t^3}{50} - t$ where θ is in radians, and t is in seconds. Calculate the displacement, the angular velocity and the acceleration at the end of 10 seconds (3mks)