TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BMCS/BSSC

AMA 4215: STATISTICS III

END OF SEMESTER EXAMINATION

SERIES: APRIL2016

TIME: 2HOURS

DATE: 20May2016

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of Choose No questions. Attempt Choose instruction.

Do not write on the question paper.

Question ONE (30 MarkS)

Question 1 (30 marks)

(a) Fatal accidents occur at random at a known 'black spot', following a Poisson process with mean 4 per year.

(i) Draw a diagram of the probability mass function of X, the actual annual number of fatal accidents. (4 marks)

(ii) Determine the probability that in a given year there is at most one fatal accident. (2 marks)

(b) The random variable X follows the exponential distribution with rate parameter λ, so that the probability density function (pdf) of X is given by

$$f(x) = \lambda e^{-\lambda x}, \;\; x > 0, \; \lambda > 0.$$

Given that the moment generating function of X, $M_X(t)$ say, is given by

$$M_X(t) = \left(1 - \frac{t}{\lambda}\right)^{-1}, \; t < \lambda,$$

show that the mean and variance of X are given by $\frac{1}{\lambda}$ and $\frac{1}{\lambda^2}$ respectively (6 marks)

(c) The bivariate probability distribution of the random variables X and Y is summarised in the following table.

		Y			
		0	1	2	3
X	0	k	$6k$	$9k$	$4k$
	1	$8k$	$18k$	$12k$	$2k$
	2	k	$6k$	$9k$	$4k$

(i) Find k. (3 marks)

(ii) Obtain the marginal distributions of X and Y. (4 marks)

(d) The distribution $f_{X,Y}(x,y) = \frac{1}{\sqrt{(2\pi)}} e^{-0.5(x^2+y^2)}$ is a bivariate normal distribution. For this distribution, state $\sigma_X, \sigma_Y, \mu_X, \mu_Y, \rho$ (4 marks)

(e) The joint density function of 2 random variables X and Y is given as

$$f_{X,Y}(x,y) = \begin{cases} \frac{2}{5}(2x+3y), & 0 < x < 1, \quad 0 < y < 1 \\ 0, & elsewhere \end{cases}$$

(i) Verify that $f_{X,Y}(x,y)$ is a pdf (4 amrks)

(ii) Find $P[(X,Y)] \in A$, where $A = \left\{(x,y) \middle| 0 < x < \frac{1}{2}, \frac{1}{4} < y < \frac{1}{2}\right\}$ (3 marks)

Question TWO (20 MarkS)

(a) Suppose X and Y are independent random variables having Poisson distributions with respective means λ (> 0) and μ (> 0).

(i) Show that $X + Y$ also follows a Poisson distribution. (5 marks)

(ii) Find $P(X = k|X + Y = n)$ (when k and n are integers with $0 \le k \le n$. For given fixed $n > 0$, name the distribution you have obtained. (7 marks)

(b) Telephone calls arriving at a computer helpline are classed as urgent or standard; urgent calls average 8 per hour, standard calls average 24 per hour. Ten calls arrive within 30 minutes; find (to two significant figures) the probability that at most two of them are urgent, stating any assumptions you make. (8 marks)

Question THREE (20 MarkS)

The continuous random variables X and Y have joint probability density function $f(x,y) = kxy$ if $0 < x < y < 1$, with $f(x,y) = 0$, elsewhere, where k is a constant.

(a) Evaluate k, and find the marginal probability densities of X and Y. Say, with a reason, whether or not X and Y are independent. (10marks)

(b) Show that, for all non-negative integers r and s, $E(X^r Y^s) = \frac{8}{(r+2)(r+s+4)}$

Hence find the correlation between X and Y. (10marks)

Question FOUR(20 Marks)

(a) Suppose X and Y are independent random variables, each following the chis-quared distribution with four degrees of freedom; this distribution has probability density function (pdf) $we^{-w/2}/4$ on $w > 0$.

Define new random variables $U = X / Y$ and $V = Y$. Obtain the joint pdf of U and V, and hence show that U has pdf $h(u)$=(6u)/(1+u)^4 for $u > .0$ (10marks)

[You may use without proof the result $\int_0^\infty t^k e^{-t} dt = k!$ when k is a positive integer.]

Name the distribution followed by U *in (a)*. Using the density function, show that $E(U) = 2$, and hence verify that $E(X / Y) > E(X) / E(Y)$. Explain briefly why this is no surprise. (10marks

Question FIVE(20 Marks)

(a) The random variables $X1$, $X2$ have the bivariate Normal distribution with expectation $u = (\mu_1 \ \ \mu_2)^T$ covariance matrix $\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$

(i) Write out explicitly the joint probability density function of $X1$ and $X2$. (3 marks)

(ii) State (without proof) the marginal distribution of $X2$ and write out its marginal probability density function. (1mark)

(iii) Hence obtain the conditional probability density function of $X1$ given that $X2 = x2$. Identify this as a Normal distribution with parameters that you should state explicitly. (6 marks)

(b) The random variables $X1$, $X2$, $X3$ have the multivariate Normal distribution with expectation $u = (\mu_1 \ \ \mu_2 \ \ \mu_3 \ \ )^T$and covariance matrix, $\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix}$. Let $\Sigma_{23} = \begin{pmatrix} \sigma_2^2 & \sigma_{23} \\ \sigma_{23} & \sigma_3^2 \end{pmatrix}$, a sub-matrix of Σ . In general, the conditional distribution of $X1$ given that $X2= x2$, $X3= x3$ is a Normal distribution with

$$E(X_1 \mid x_2,\, x_3) = \mu_1 + (\sigma_{12} \quad \sigma_{13})\Sigma_{23}^{-1}\begin{pmatrix} x_2 - \mu_2 \\ x_3 - \mu_3 \end{pmatrix},$$

$$\mathrm{Var}(X_1 \mid x_2,\, x_3) = \sigma_1^2 - (\sigma_{12} \quad \sigma_{13})\Sigma_{23}^{-1}\begin{pmatrix} \sigma_{12} \\ \sigma_{13} \end{pmatrix}.$$

Obtain the parameters of the conditional distribution of $X1$ given that $X2= x2$, $X3= x3$ in the special case where $X2$ and $X3$ are independent random variables. Find an expression for the multiple correlation of $X1$ on both $X2$ and $X3$ in this case. (10 marks)