TECHNICAL UNIVERSITY OF MOMBASA

Faculty of applied and Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE

AMA 4103:DISCRETE MATHS

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: 2016

PAPER A

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of 5 questions. Question one is compulsory. Answer any other two questions

Do not write on the question paper.

SECTION A

Question one

(1)(a) Define the following notions (i) Recurrence relation (ii) Modus Tollens(iii) Hypothetical syllogism

(iv) Involution (v) Boolean algebra. (5mks)

(b) (i) Test the validity of the following argument "if I leave college then I will get a job in a bank. Iam not leaving college so I won't get a job in a bank ". (2mks)

(ii) Determine the truth table for the compound statement $(p \vee \neg q) \wedge \neg p$. (2mks)

(c) Let $\mathrm{A} = \{1,2,3,4\}$ *and* $R = \{(1,3),(4,2),(2,4),(2,3),(3,1)\}$ .Test whether the relation

Is reflexive or not. (2mks)

(d) Let $\mathrm{A} = \{0,1,2\}$ *and* $B = \{a,b\}$ .Find the Cartesian product of A and B. (2mks)

(e) $\forall\, a,b,c \in \mathrm{B}$ where B is a Boolean algebra show that

(i) $a + a = a$ (2mks)

(ii) $a + a \bullet b = a \quad and \quad a \bullet (a+b) = a$ (2mks)

(iii) $\overline{a+b} = \bar{a} + \bar{b} \quad and \quad \overline{a \bullet b} = \bar{a} + \bar{b}$. (2mks)

(f) Let $\mathrm{A} = \{1,2,3\}$. Find the power set of A (2mks)

(g(i)) State and prove Demorgan's laws (2mks)

(ii) Show that $\mathrm{A} - (B - C) = (A - B) \cup (A - C)$. (2mks)

(h) Prove that for every integer n, if n is even then the square of n is also even. (2mks)

(i) prove that that $\sqrt{2}$ is irrational . (2mks)

SECTION B

Question two

(2)(a)L ist two significance of learning Boolean algebra to computer scientists. (2mks)

(b) Let $f : \mathbb{R} \to \mathbb{R} \quad and \quad g : \mathbb{R} \to \mathbb{R}$ defined by. $f(x) = x + 4 \quad and \quad g(x) = \dfrac{1}{(x^2 + 2)}$

Find (i) $g \circ f$. (3mks)

(ii) $f^{-1}(x(x))\ and\ g^{-1}(x)$ (3mks)

(iii) $f^{-1} \circ g^{-1}$. (3 mks)

(c) Show that $f : \mathbb{R} - \{1\} \rightarrow \mathbb{R} - \{3\}$ defined by $f(x) = \dfrac{3x}{x-1}$ is a bijection. (6mks)

(d) Find the sequence of the recurrence relation $a_r = a_{r-1} + a_{r-2}$. (3mks)

Question three ,(20MKS)

(3)(a) Prove by contradiction that if $x + y > 5\ then\ either\ x < 2\ or\ x > 3$. (5mks)

(b) Test whether $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)\ and\ p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$ are

Logical equivalences. (6mks)

(c) (c) A boat which carries ten people, carries a group of 13 men and 7 women across a river.

Find the number of ways in which the group may be taken across if all women go on the first

trip. (3mks)

(d) Draw and show the number of labeled graphs with three vertices. (3mks)

(e) (j) Use mathematical induction to prove that
$1^2 + 2^2 + \ldots\ldots\ldots\ldots. + n^2 = \frac{1}{6}\, n(n+1)(2n+1)$ (3 marks).

Question four(20MKS)

(4) Of the cars sold during during july, 90 had air conditioning, 100 had automatic transmissions,75 had power steering, 5 cars had all the three, 20 cars had none of these extras, 20 cars had only air conditioning, 60 cars had only automatic transmission, 30cars had only power steering, 10 cars had both automatic transmission and power steering.

(i)How many cars had both power steering and air conditioning. (3mks)

(ii) How many cars had both automatic transmission and air conditioning. (3mks)

(iii) How many had neither power steering nor automatic transmission. (3mks)

(iv) How many cars were sold in july. (3mks)

(v) How many had either of the three extras. (3mks)

(b) Let A be the adjacency matrix of a graph G with n vertices,show that the ij^{th} entry of

The matrix gives the number of walks of length l from vertex v_i *to* v_j. (5mks)

QUESTION FIVE (20MKS)

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(5)(a) Find the solution of the recurrence relation

$a_r = a_{r-1} + 2a_{r-2}$ *where* $a_0 = 2, a_1 = 7$. (7mks)

(b) Find the particular and total solution of the recurrence relation

$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1$. (8mks)

(c) Draw and show the number of labeled graphs with three vertices. (5mks)