TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR THE SECOND SEMESTER IN THE THIRD YEAR OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE/ BACHELOR OF SCIENCE IN STATISTICS AND COMPUTER

MAY 2016 SERIES EXAMINATION

UNIT CODE: AMA 4319

UNIT TITLE: TEST OF HYPOTHESIS

TIME ALLOWED: 2HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS)

a) Define the following terms as used in hypothesis testing;

i. Simple and composite hypothesis

ii. Probability value (p-value)

iii. Type I and Type II error

(5 marks)

b) A tire company wants to change the tire design. Economically the modification can be justified if average life time with the new design exceeds 20,000 miles. A random sample of size n=16 new tires is tested. Assume lifetime are N $(\mu, 1500^2)$. The tires yield $\overline{x} = 20{,}758$.

i. Should the new design be adopted? Test at $\alpha = 0.01$

ii. Find the probability of not rejecting the null hypothesis when μ =21,000.

iii. Find the sample size needed to have $\beta(21{,}000) = 0.1 \;\; where \; \beta(21{,}000)$ is the probability of not rejecting the null hypothesis when $\mu = 21{,}000.$

(10 marks)

c) Suppose the mean weight of broilers in kenchic poultry farm was found to be 5.2 kg. Assume that the mean population weight is 5.4 kg and the population standard deviation is 0.6 kg. At 0.05 significance level, what is the probability of having type II for a sample of 9 broilers. (6 marks)

d) Consider a random sample $x_1, x_2, \ldots \ldots \ldots \ldots, x_n$ from a distribution having the probability density function $f(x) = \theta e^{-\theta x} \;\; , x \geq 0$. Show that the best critical region for testing $H_0 : \theta = \theta_0 \; against \; H_1 : \theta = \theta_1$ is given by $\{(x_1, x_2, \ldots \ldots, x_n) : \sum x_i \geq C\}$. Assume $\theta_1 < \theta_0$ (5 marks)

e) A sample of 40 sales receipts from a grocery store has a mean $\overline{x}$ =137 and standard deviation $= 30.2$. Use these values to test whether or not the mean sales at the grocery store are different from 150 at 0.01 significance level

(4 marks)

<u>QUESTION TWO (20 MARKS)</u>

a) (i) State the Neyman-Pearson Lemma

(ii) Let $x_1, x_{2, \ldots\ldots\ldots}, x_n$ denote a random sample from a $N(\mu, 36)$. Show that, according to the Neyman-Pearson lemma, $C = \{(x_1, x_{2, \ldots\ldots\ldots}, x_n) : \overline{x} \geq k\}$ for some constant k , is the best critical region for testing,

$H_0 : \mu = 50 \;\; against \;\; H_1 : \mu = 55$ (9 marks)

(iii) If the sample size is n=16, determine the value of k so that critical region of this test is of size $\alpha = 0.05$ (3 marks)

b) Let X be a random variable which is $N(\mu, 64)$ distributed. A random sample of size n=36 is chosen from this population and the critical region defined by $C = \{\underline{x} : \overline{x} > 52\}$ is used to test the hypothesis

$H_0 : \mu = 50 \ against \ H_1 > 50,$ find

i. The size of the test

ii. The power of the test

iii. The probability of type II error when the sample mean $\overline{x} = 55$

(8 marks)

QUESTION THREE (20 MARKS)

a) Let Ω denote the total parameter space, ω a subset of Ω and ω' the complement of ω with respect to Ω.

i. Define the critical region for the likelihood ratio test for

$$H_0: \theta \in \omega \ against \ H_1 : \theta \in \omega'$$ (4 marks)

ii. Assuming that X is $N(\mu, 5)$, show that the likelihood ratio critical region for testing

$$H_0 : \mu = 59 \ against \ H_1 : \mu \neq 59 \ is \ C = \left\{ \overline{x} : \frac{|\overline{x} - 59|}{\sigma/\sqrt{n}} \geq k \right\}$$

(8 marks)

b) Let X be $N(\mu, 225)$.

i. To test $H_0: \mu = 59 \ against \ H_1: \mu \neq 59$, give the critical region of size $\alpha = 0.05$ specified by the likelihood ratio test criterion.

ii. If a sample of size n=100 yields $\overline{x} = 56.13$ is the null hypothesis not rejected? What is the p-value for this test?

(8 marks)

QUESTION FOUR (20 MARKS)

a) Define a P-value a test. (2 marks)

b) Distinguish between:

I. one tailed test and a two tailed test as used in hypothesis testing (4 marks)

ii. most powerful test and a uniformly most powerful test . (4 marks)

c). Let X denote the height of a randomly chosen university students. Assume X is normally distributed with unknown mean μ and standard deviation of 9. Take a random sample of n=25 students, so that, after setting the probability of committing type I error at $\alpha = 0.05$, test the null hypothesis $H_0 : \mu = 170$ against the alternative hypothesis that $H_1 : \mu > 170$.

i. What is the power of the hypothesis test if the true population mean were $\mu = 175$?

ii. Find the sample size that is necessary to achieve 0.90 power at the alternative $\mu = 175$.

(10 marks)

QUESTION FIVE (20 MARKS)

a) If we want to test the null hypothesis that the mean μ of normal population with variance $\sigma^2 = 1$ if $H_0: \mu = \mu_0$ is against an alternative $H_1: \mu = \mu_1$ where $\mu_1 > \mu_0$. Find the value of k that provides a critical region of size $\alpha = 0.05$ for a sample of size n. (10 marks)

b) Suppose $X_1, X_{2,.................,} X_n$ is a random sample from a normal population with μ and variance 16. Find the test with the best critical region, that is, find the most powerful test with a sample of size n=16 and significance level $\alpha = 0.05$ to test the simple null hypothesis $H_0: \mu = 10 \ against \ H_1 = 15.$ (10 marks)