TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR THE SECOND SEMESTER IN THE THIRD YEAR OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE/ BACHELOR OF SCIENCE IN STATISTICS AND COMPUTER

MAY 2016 SERIES EXAMINATION

UNIT CODE: AMA 4319

UNIT TITLE: TEST OF HYPOTHESIS

TIME ALLOWED: 2HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS)

1. a) Define the following terms as used in hypothesis testing
   - i. Type I error
   - ii. Level of significance
   - iii. Test statistic
   - iv. P-value

(8 marks)

b) It is suspected that a coin is no balanced (not fair). Let p be the probability of getting a head. To test $H_0 : P = 0.5$ against the alternative hypothesis $H_1 : P > 0.5$, a coin is tossed 15 times. Let Y equal the number of times a head is observed in 15 tosses of this coin. Assume the rejection region to be $\{Y \geq 10\}$. Find:

i. the probability of Type I error (5 marks)

ii. the probability of Type II error when $P = 0.7$ (3 marks)

iii. the rejection region of the form $\{Y \geq K\}$ for $\alpha = 0.01$ (3 marks)

c) Consider a random sample chosen from a normal population with $\sigma = 3.1$ being its true standard deviation. Determine how large a sample size should be for testing $H_0 : \mu = 5$ $against$ $H_1 : \mu = 5.5,$ $in\ order\ that$ $\alpha = 0.01$ $and\ \beta = 0.05$ (5 marks)

d) Suppose we want to test the null hypothesis that the mean μ of normal population with variance $\sigma^2 = 1$ if μ_0 is against an alternative μ_1 $where$ $\mu_1 > \mu_0$. Find the value of K such that $\overline{X} > k$ provides a critical region of size $\alpha = 0.05$ for a sample of size n. (6 marks)

<u>QUESTION TWO (20 MARKS)</u>

a) Define a rejection region of a test. (2 marks)

b) Distinguish between the following concepts as used in hypothesis testing

i. a one tailed test and a two tailed test. (4 marks)

ii. a most powerful test and a uniformly most powerful test . (4 marks)

c) The management of a local health club claims that its members lose on the average 15 pounds or more within the first 3 months after joining the club. To check this claim, a consumer agency took a random sample of 45 members of this health club and found that they lost an average of 13.8 pounds within the first 3 months of membership, with a sample standard deviation of 4.2 pounds.

i. Find the p – value of this test . (8 marks)

ii. Based on the p-value in (i) would you reject the null hypothesis at $\alpha = 0.01$? (2 marks)

QUESTION THREE (20 MARKS)

a) State the generalized likelihood ratio test . (4 marks)

b) Let $X_1, X_2, \dots \dots \dots X_n$ be a random sample from an $N(\mu, \sigma^2)$. Assume that σ^2 is unknown. We wish to test , at level α , $H_0 : \mu = \mu_0 \;\; vs. \; H_1 : \mu \neq \mu_0$. Find an appropriate likelihood ratio test.

(16 marks)

QUESTION FOUR (20 MARKS)

a) Let $X_1, X_2, \dots \dots \dots \dots \dots., X_n$ be a random sample from a normal distribution with a known mean μ and variance σ^2 =1 .Test the hypothesis that :

$$H_0 : \mu = \mu_0 \;\; against \;\; H_1 : \mu > \mu_0$$

(10 marks)

b) Suppose X is a random sample from a normal population with mean μ and variance 16. Taking a sample of size n=16 find the most powerful test with significance level $\alpha = 0.05$,test the hypothesis $H_0: \mu = 10 \;\; ahainst \;\; H_1: \mu = 15.$

(10 marks)

QUESTION FIVE (20 MARKS)

a) Let $X_{1,} X_{2,\dots\dots\dots\dots} X_n$ be a random sample from a normal distribution unknown mean μ. Test the hypothesis $H_0 : \sigma^2 = \sigma_0^2 \;\; against \;\; H_1 : \sigma^2 \neq \sigma_0^2$. (15 marks)

b) In a random sample of 19 babies of a certain age, the standard deviation of their weights was 2.5 kg. Test the hypothesis at $\alpha = 0.01$ that

$$H_0 : \sigma = 3 \;\; against \;\; H_1 : \sigma \neq 3$$

(5 marks)