# TECHNICAL UNIVERSITY OF MOMBASA

*A Centre of Excellence*

*Faculty of Applied & Health Sciences*

## DEPARTMENT OF MATHEMATICS AND PHYSICS

## MAY 2016 SERIES EXAMINATION

## UNIT CODE: AMA 4421 UNIT TITLE:NUMERICAL ANALYSIS II

## SUPPLIMENTARY EXAMINATION

## TIME ALLOWED: 2HOURS

***INSTRUCTIONTO CANDIDATES:***

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

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## QUESTION ONE (30 MARKS) COMPULSORY

a. Find the first, second and third derivatives of the function tabled below, at the point x=1.5

| X | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|
| F(x) | 3.375 | 7.0 | 13.625 | 24 | 38.875 | 59 |

(5 marks)

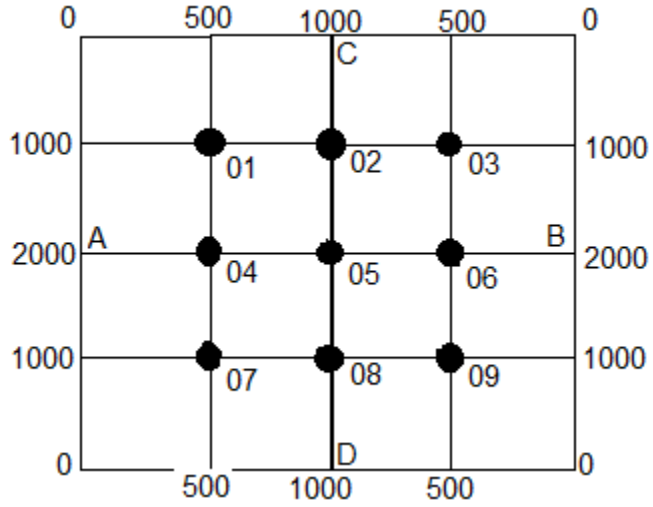
b. Let $A = \begin{pmatrix} 12 & -15 \\ 4 & -4 \end{pmatrix}$ be a square matrix

i. Write down the characteristic equation of A (2 marks)

ii. Calculate the eigen values of A (2 marks)

iii. Determine the eigen vectors of A (2 marks)

c. Solve the elliptic equation for the following square mesh with boundary values as shown




(8 marks)

d. The speed, v meter per second of a car, t seconds after it starts is shown in the following table.

| T | 0 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| v | 0 | 3.6 | 10.08 | 18.9 | 21.6 | 18.54 | 10.26 | 5.4 | 4.5 | 5.4 | 9 |

Using Simpson's $\frac{1}{3}$rule, find the distance travelled by the car in 2 minutes? (5 marks)

e. Solve the system of equations by using Gauss-Jordan elimination method

$x + 2y + z = 8$

$2x + 3y + 4z = 20$

$4x + 3y + 2z = 16$

(6 marks)

## QUESTION TWO (20 MARKS)

a. Use Trapezoidal rule to evaluate $\int_0^1 x^3 dx$ considering five sub intervals (5 marks)

b. Using crank Nicholson's method, solve $U_{xx} = 16U_t \quad 0 < x < 1 \;\; t > o \;\; U(x,0) = 0, U(0.t) = 0 \; and \; U(1,t) = 50t$. compute u for two steps in t direction taking $h = \frac{1}{4}$

(9 marks)

c. Use Gaussian elimination to solve

$x + y + z = 3$

$x + 2y + 3z = 4$

$x + 4y + 9z = 6$ (6 marks)

## QUESTION THREE (20 MARKS)

a. Maximize $P = x + 4y$ subject to

$-x + 2y \leq 6$

$5x + 4y \leq 40$

$x, y \geq 0$ (5 marks)

b. By the methods of least squares, find the straight line that best fits the following data

| X | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| y | 14 | 27 | 40 | 55 | 68 |

(6 marks)

c. Solve the poisson equation

$\nabla^2 U = -10(x^2 + y^2 + 10)$

over the square mesh with sides $x = 0, y = 0, x = 3, y = 3\ with\ u = 0$ on the boundary and mesh length=1 (9 marks)

## QUESTION FOUR (20 MARKS)

a. Solve the Laplace equation $U_{xx} + U_{yy} = 0$ inside the square region bounded by the lines x=0, x=4, y=0, y=4 given that $U = x^2y^2$ on the boundary. Use relaxation technique (7 marks)

b. solve the system of equations

$x + y + z = 3$

$x + 2y + 3z = 4$

$x + 4y + 9z = 6$

by Crout's methods (8 marks)

c. find the orthogonal trajectories of the family of curves,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$$

Where $\lambda$ is a parameter (5 marks)

## QUESTION FIVE (20 MARKS)

a. Solve the following system, by the method of triangularisation

$2x - 3y + 10z = 3$

$-x + 4y + 2z = 20$

$5x + 2y + z = -12$ (8 marks)

b. Find the numerical value of the first derivative at x=0.4 of the function f(x) defined below

| X | 0.1 | 0.2 | 0.3 | 0.4 |
|---|---|---|---|---|
| F(x) | 1.10517 | 1.22140 | 1.34986 | 1.49182 |

(7 marks)

c. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $^3/_8$ rule taking h= $^1/_6$ (5 marks)

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