



TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

MAY 2016 SERIES EXAMINATION

UNIT CODE: AMA 4421 UNIT TITLE: NUMERICAL ANALYSIS

II

SUPPLEMENTARY EXAMINATION

TIME ALLOWED: 2HOURS

INSTRUCTION TO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

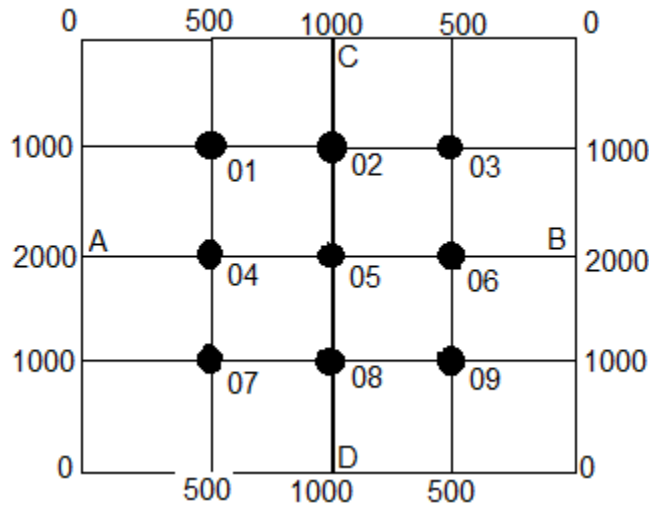
QUESTION ONE (30 MARKS) COMPULSORY

- a. Find the first, second and third derivatives of the function tabled below, at the point $x=1.5$

X	1.5	2.0	2.5	3.0	3.5	4.0
F(x)	3.375	7.0	13.625	24	38.875	59

(5 marks)

- b. Let $A = \begin{pmatrix} 12 & -15 \\ 4 & -4 \end{pmatrix}$ be a square matrix
- Write down the characteristic equation of A (2 marks)
 - Calculate the eigen values of A (2 marks)
 - Determine the eigen vectors of A (2 marks)
- c. Solve the elliptic equation for the following square mesh with boundary values as shown



(8 marks)

- d. The speed, v meter per second of a car, t seconds after it starts is shown in the following table.

T	0	12	24	36	48	60	72	84	96	108	120
v	0	3.6	10.08	18.9	21.6	18.54	10.26	5.4	4.5	5.4	9

Using Simpson's $\frac{1}{3}$ rule, find the distance travelled by the car in 2 minutes? (5 marks)

- e. Solve the system of equations by using Gauss-Jordan elimination method

$$x + 2y + z = 8$$

$$2x + 3y + 4z = 20$$

$$4x + 3y + 2z = 16$$

(6 marks)

QUESTION TWO (20 MARKS)

- a. Use Trapezoidal rule to evaluate $\int_0^1 x^3 dx$ considering five sub intervals (5 marks)

- b. Using crank Nicholson's method, solve $U_{xx} = 16U_t$ $0 < x < 1$ $t > 0$ $U(x, 0) = 0$, $U(0, t) = 0$ and $U(1, t) = 50t$. compute u for two steps in t direction taking $h = \frac{1}{4}$

(9 marks)

- c. Use Gaussian elimination to solve

$$x + y + z = 3$$

$$x + 2y + 3z = 4$$

$$x + 4y + 9z = 6$$

(6 marks)

QUESTION THREE (20 MARKS)

- a. Maximize $P = x + 4y$ subject to

$$-x + 2y \leq 6$$

$$5x + 4y \leq 40$$

$$x, y \geq 0$$

(5 marks)

- b. By the methods of least squares, find the straight line that best fits the following data

X	1	2	3	4	5
y	14	27	40	55	68

(6 marks)

- c. Solve the poisson equation

$$\nabla^2 U = -10(x^2 + y^2 + 10)$$

over the square mesh with sides $x = 0, y = 0, x = 3, y = 3$ with $u = 0$ on the boundary and mesh length=1

(9 marks)

QUESTION FOUR (20 MARKS)

- a. Solve the Laplace equation $U_{xx} + U_{yy} = 0$ inside the square region bounded by the lines $x=0, x=4, y=0, y=4$ given that $U = x^2y^2$ on the boundary. Use relaxation technique

(7 marks)

- b. solve the system of equations

$$x + y + z = 3$$

$$x + 2y + 3z = 4$$

$$x + 4y + 9z = 6$$

by Crout's methods

(8 marks)

- c. find the orthogonal trajectories of the family of curves,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$$

Where λ is a parameter

(5 marks)

QUESTION FIVE (20 MARKS)

- a. Solve the following system, by the method of triangularisation

$$2x - 3y + 10z = 3$$

$$-x + 4y + 2z = 20$$

$$5x + 2y + z = -12$$

(8 marks)

- b. Find the numerical value of the first derivative at $x=0.4$ of the function $f(x)$ defined below

X	0.1	0.2	0.3	0.4
F(x)	1.10517	1.22140	1.34986	1.49182

(7 marks)

- c. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $3/8$ rule taking $h=1/6$

(5 marks)

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