TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR THE SECOND SEMESTER IN THE FOURTH YEAR OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

MAY 2016 SERIES EXAMINATION

UNIT CODE: AMA 4426

UNIT TITLE: STOCHASTIC PROCESSES

TIME ALLOWED: 2HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS)

(a) Define the following:

(i) A stochastic process (2 marks)

(ii) A Bernoulli process (2 marks)

(b) Let Y have a geometric distribution given by

$$P(Y = k) = \begin{cases} q^k p; & k = 0,1,2,3, \ldots . . \\ 0 \; ; & elsewhere \end{cases}$$

Find (i) the probability generating function of Y (4 marks)

(ii) the mean and variance of Y (6 marks)

(c) . Let $\{X_n : n \geq 0\}$ be a Markov chain with three states 0,1,2 and transition probability matrix

$$\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$$

And the initial probability distribution $P(X_0 = i) = \begin{cases} \frac{1}{4} , i = 0 \\ \frac{1}{3} , i = 1 \\ \frac{5}{12} , i = 2 \end{cases}$

Find :

(i) $P(X_2 = 2 , \; X_1 = 1 / X_0 = 2)$ (3marks)

(ii) $P \, (X_1 = 1 / X_0 = 2)$ (1 mark)

(iii) $P \, (X_2 = 2 \, / \, X_1 = 1)$ (1 mark)

(iv) $P \, (X_3 = 1 , \; X_2 = 2 , X_1 = 1 , X_0 = 2)$ (3 marks) (d). The joint distribution of two random variables X and Y is given by:

$$P_{jk} = P\,\{X = j\,, Y = k\} = \begin{cases} q^{j+k}p^2\,, & j = 0,1,2,\ldots\,, k = 0,1,2,\ldots, \quad p + q = 1 \\ 0 & otherwise \end{cases}$$

Obtain the:

(i). bivariate p.g.f of X and Y (4 marks)

(ii). P.g.f of X (2 marks)

(iii). P.g.f of X+Y (2 marks)

QUESTION TWO (20 MARKS)

(a) Define the following terms :

(i) Irreducible Markov chain (2 marks)

(ii) Persistent state (2 marks)

(iii) A periodic state (1 mark)

(iv) Ergodic state (1 mark)

(b). A markov chain with state space $\{E_1, E_2, E_3\}$ has the following probability transition matrix

$$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

Classify the states of the process. (14 marks)

QUESTION THREE (20 MARKS)

Consider a population whose size at time t is Z(t) and let the probability that the population size is n be denoted by $P_n(t) = P\{Z(t) = n\}$ with $P_1(0) = 1$ and $P_n(0) = 0$, $n \neq 1$. Further let :

(i) The chance that an individual produces a new member in time t interval Δt be λΔt where λ is some constant be n.

(ii) The chance of an individual producing more than one member be 0(Δt) (i.e negligible).

(a) Show that $P_n(t) = e^{-\lambda t}\{1 - e^{-\lambda t}\}^{n-1}$, $n \neq 1$

(b) Find the second raw moment of the process

(20 marks)

QUESTION FOUR (20 MARKS)

(a) Explain the following terms:

(i) A strictly stationary stochastic process (2 marks)

(ii) A covariance stationary process (2 marks)

(iii) An evolutionary process (2 marks)

b) (i) Write down the differential-difference equations for the Polya process. Hence obtain the probability generating function given that $P_n(0) = 1$ $when\, n = 0$ and $P_n(0) = 0$ $when\, n \neq 0$

(ii) Show that the Polya process is not covariance stationary.

(14 marks)

QUESTION FIVE (20 MARKS)

Consider the difference-differential equations for the Poisson process given by

$$P'_n(t) = \begin{cases} -\lambda P_n(t) + \lambda P_{n-1}(t) & : \ n \geq 1 \\ -\lambda P_0(t) & : \ n = 0 \end{cases}$$

With initial conditions $P_0(0) = 1 \; when \; n = 0 \; and \; P_n(0) = 0 \; when \; n \neq 0$

(i) Find the solution of the equation.

(ii) Use Feller's method to find the mean and variance of the process.

(20 marks)