



TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

MATHS AND PHYSICS DEPARTMENT

UNIVERSITY EXAMINATION FOR: BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

AMA 4435: MEASURE INTEGRATION AND PROBABILITY PAPER 1

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: MAY 2016

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of FIVE questions. Attempt QUESTION 1 AND ANY OTHER TWO FROM QUESTIONS 2- 5.

Do not write on the question paper.

Question ONE (30 MARKS)

- a. Define the following terms
 - I. Measure of a set (2 marks)
 - II. A space (2 marks)
 - III. A complete measure (2 marks)
- b. Distinguish between the positive and negative parts of a function (4marks)
- c. Let (X, \mathfrak{X}) be a measurable space, if $\mathfrak{x} \subseteq \mathfrak{X}$ is called a σ - algebra, outline the conditions that it must be satisfy.(4 marks)
- d. By use of a counter example, show that if $f \subset \mathfrak{x}$, so that if and f and $f \in \mathfrak{x}$ but the converse is not true (6 marks)
- e. State the monotone convergence theorem(4 marks)
- f. State three examples of measurable spaces (3 marks)
- g. Outline three conditions that make a measurable set to be countable.(3marks)

Question TWO (20 marks)

- a. Let (X, \mathfrak{X}) be a measurable space. In order that a function $f: X \rightarrow \mathbb{R}_e$ be \mathfrak{X} -measurable. Outline the necessary and sufficient conditions that must be fulfilled (8marks)
- b. Let (X, \mathfrak{X}) be a measurable and $f, g: X \rightarrow \mathbb{R}_e$ be \mathfrak{X} -measurable functions and let $c \in \mathbb{R}$. Prove that the functions $cf, c + f, f^2, |f|, f + g, fg, f^+$ and f^- are all \mathfrak{X} -measurable (12 marks)

Question THREE

- a. Let $f: X \rightarrow \mathbb{R}_e$ be measurable with $f \gg 0$ ($f: X \rightarrow (0, \infty)$). Prove that there exists an increasing sequence of simple functions $\phi_n: X \rightarrow \mathbb{R}$ which converges to f pointwise. i.e. $f(x) = \lim_{n \rightarrow \infty} (\phi_n(x))$ for all $x \in X$ (12 marks)
- b. Prove that if a function f is measurable then a measurable function is integrable iff $|f|$ is integrable and $|\int f d\mu| \ll \int |f d\mu|$ (8 marks)

Question FOUR (20 marks)

- a. Let ψ be a simple measurable function belonging to $M^+(X, \mathfrak{X})$. Let $\lambda: \mathfrak{X} \rightarrow \mathbb{R}_e$ be defined by $\lambda(E) = \int \psi \chi_E d\mu$ for $E \in \mathfrak{X}$. Show that $\lambda(E)$ is a measure (14 marks)
- b. State Fatou's lemma. Do not prove it. (4 marks)
- c. State the law of large numbers.

Question FIVE (20 marks)

- a. What do you understand by the term probability measure? State four points. (4 marks)
- b. State Demorgan's laws and prove that a set is enclosed under countable intersections (6 marks)
- c. Let (X, \mathfrak{X}, μ) be a measure space. prove that μ is monotone if $E, F \in \mathfrak{X}$ and $F \subset E$ (4 Marks)
- d. State and prove the central limit theorem (6marks)