



TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

AMA 5106: TEST OF HYPOTHESIS

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 3 HOURS

DATE: MAY

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of five questions. Attempt QUESTION ONE and any other TWO.

Do not write on the question paper.

Question ONE

- a. State and prove Neyman-Pearson Lemma (8 marks)
- b. A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed, with standard deviation 0.25 Volts, and the manufacturer wishes to test $H_0; \mu = 5$ Volts against $H_1; \mu \neq 5$ Volts, using 8 units.
 - i. The acceptance region is $4.85 \leq \bar{x} \leq 5.15$ Find the level of significance. (4marks)
 - ii. Find the power of the test for detecting a true mean output voltage of 5.1 Volts. (5marks)
- c. Show that the class of all test functions is a convex function (3marks)
- d. Define the power function of a test (4marks)
- e. Show that 1-parameter exponential family has a monotone likelihood ratio. (6marks)

Question TWO

- a. Let x be a random variable with probability density function $f(x)$. Find a size α test for;
(7marks)

$$H_0; f(x) = f_0(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$H_1; f(x) = f_1(x) = \frac{1}{\pi} \frac{1}{1+x^2}$$

- b. Let x_1, x_2, \dots, x_n be independently identically distributed $N(0, \sigma^2)$ random variables. Determine whether there exists a uniform most powerful test for the hypothesis of the form $H_0; \sigma^2 = \sigma_0^2$ against $H_1; \sigma^2 = \sigma_1^2$ (8 marks)
- c. Show that for testing $H_0; \theta_1 \leq \theta \leq \theta_2$ against $H_1; \theta < \theta_1$ or $\theta > \theta_2$ there exists a uniform most powerful unbiased size α test given by $\phi(x) = \begin{cases} 1 & \text{if } T(x) > c_1 \\ v & \text{if } T(x) = c_2 \\ 0 & \text{if } c_1 < T(x) < c_2 \end{cases}$ (5 marks)

Question THREE

- a. Define an unbiased test (5 marks)
- b. If the *pdf* $f(x; \theta)$ are such that the power function of every test is continuous and if ϕ_0 is uniform most powerful among all tests satisfying some conditions and is level α test, then show that ϕ_0 is unbiased. (5 marks)
- c. Let $X \sim \text{bin}(n, p)$, find an unbiased size α test for $H_0; p = p_0$ against $H_1; p = p_1$ (10 marks)

Question FOUR

- a. Let x_1, x_2, \dots, x_n be independently identically distributed $N(\mu, \sigma^2)$ random variables, Let y_1, y_2, \dots, y_n be independently identically distributed $N(\mu, \sigma^2)$ random variables. Where σ^2 is common. Suppose X'_i 's and Y'_i 's are independent. Determine a size α LRT test for $H_0; \mu = \mu_0$ against $H_1; \mu \neq \mu_0$ (10 marks)
- b. Let $x_{i1}, x_{i2}, \dots, x_{in}$ be independent normally distributed random variables with mean μ_i and variance σ_i^2 . Determine a α likelihood ratio test for the hypothesis of the form $H_0; \sigma_i^2 = \sigma_j^2$ against $H_1; \sigma_i^2 \neq \sigma_j^2$ (10 marks)

Question FIVE

- a. Determine a α likelihood ratio test for the hypothesis of the form $H_0; \sigma^2 = \sigma_0^2$ against $H_1; \sigma^2 = \sigma_1^2$ (μ is unknown) (10marks)
- b. Let y_1, y_2, \dots, y_n be independently identically distributed $N(\beta, \theta^2)$ random variables. Find a size α likelihood ratio test for testing $H_0; \beta = \beta_0$ against $H_1; \beta \neq \beta_0$ (10 marks)