Faculty of Engineering and Technology
Department of Mechanical & Automotive Engineering
UNIVERSITY EXAMINATION FOR:
BSc. Mechanical Engineering
EMG 2405 : Control Engineering I
END OF SEMESTER EXAMINATION
SERIES: APRIL 2016
TIME: 2 HOURS
DATE: Pick Date **Apr 2016**

Instruction to Candidates:

You should have the following for this examination

- *Answer booklet*
- *Non-Programmable scientific calculator*

This paper consists of **FIVE** questions. Attempt question **ONE** and any other **TWO** questions.

Maximum marks for each part of a question are as shown.

Do not write on the question paper.

Question ONE (Compulsory)

a. Figure Q1a shows a process which is being controlled in a closed-loop system with unity feedback. The process can be modelled using a first-order model with system gain, *K*, and a system time constant, *τ*. **(15 marks)**
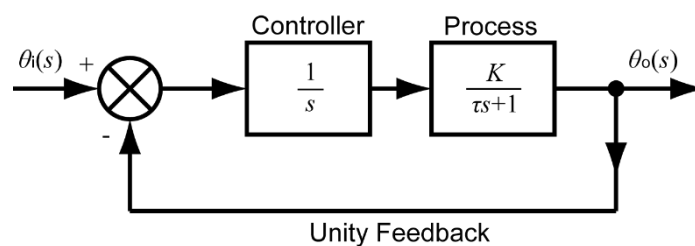

Figure Q1a

i. Obtain the expression for the forward-path transfer function for the system, $G_o(s)$.

ii. Determine the system type number.

iii. Obtain the expression for the closed-loop transfer function for the system, *G(s)*.

iv. If this system was subject to a unit step reference input, what would be the steady-state error value? Use your knowledge of system types, controller effects or the final value theorem.

v. If the system time constant, τ is 0.1 seconds and the system gain K is 2.5, will the closed-loop system be underdamped, critically damped or over-damped?

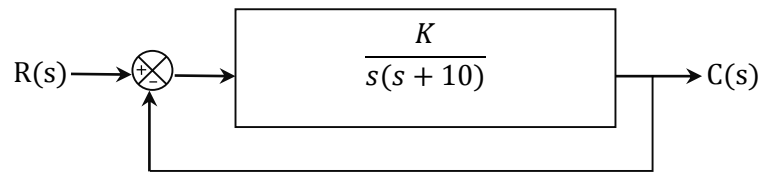b. Consider the closed loop system with a unity feedback as shown in Figure Q1b.

$$R(s) \longrightarrow \otimes \longrightarrow \boxed{\frac{K}{s(s+10)}} \longrightarrow C(s)$$

Figure Q1b

   i. Determine the gain K so that the system will have a damping ratio of 0.5.

   ii. For the obtained value of K determine the following for a unit step input:

       i. Settling time,

       ii. Rise time,

       iii. Time to peak,

       iv. Maximum overshoot   **(15 marks)**

Question TWO

a. State any TWO advantages of modern control approach to system modeling as compared to classical approach.   **(2 marks)**

b. Define the following terms:

   i. State vector,

   ii. State variable,   **(2 marks)**

c. Consider a transfer function of a system as shown in Figure Q2a

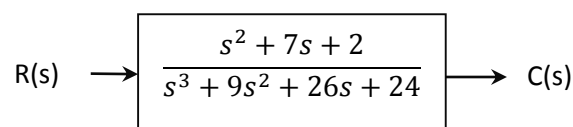$$R(s) \longrightarrow \boxed{\frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24}} \longrightarrow C(s)$$

Figure Q2a

   i. Find the state equation and output equation for the phase variable representation of the transfer function.

   ii. Draw an equivalent block diagram showing phase variables.   **(10 marks)**

d. Figure Q2b shows the translational mechanical system. Obtain the state model of the system.   **(6 marks)**
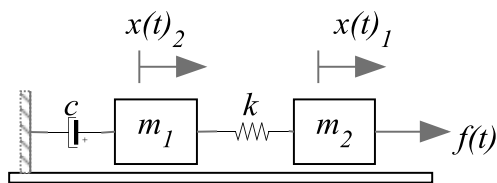
Figure Q2b

Question THREE

a.

i. State Routh-Hurwitz criteria for stability. **(3 marks)**

ii. State any THREE limitations of using Hurwitz criteria for determining stability of a system. **(6 marks)**

iii. Consider the closed loop system shown in Figure Q3a. Determine the range of values of K for which the system is stable. **(6 marks)**

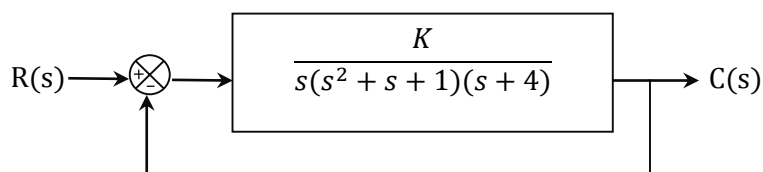$$R(s) \longrightarrow \bigotimes \longrightarrow \boxed{\dfrac{K}{s(s^2 + s + 1)(s + 4)}} \longrightarrow C(s)$$

Figure Q3a

b. Figure Q3(b) shows the block diagram for a system which has two subsystem blocks on the feed-forward path and two subsystem blocks on the negative feedback path. Derive an expression for the output response $\theta_o(s)$ in terms of the reference input $\theta_i(s)$ and the two disturbance inputs $\theta_{d1}(s)$ and $\theta_{d2}(s)$. **(5 marks)**
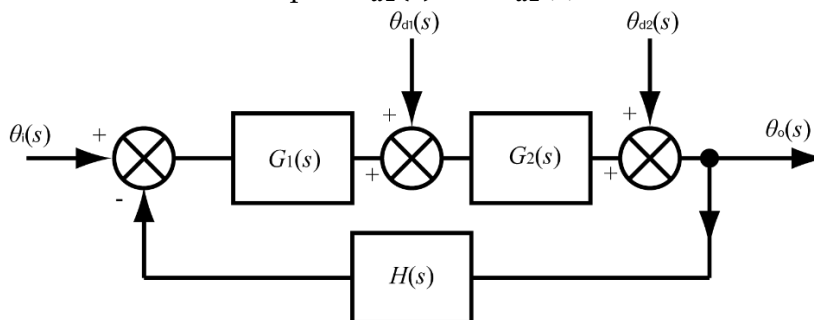

Figure Q3b

Question FOUR

Figure Q4 shows the block diagram of a complex system which includes three negative feedback loops, the outermost of which is a unity feedback loop.
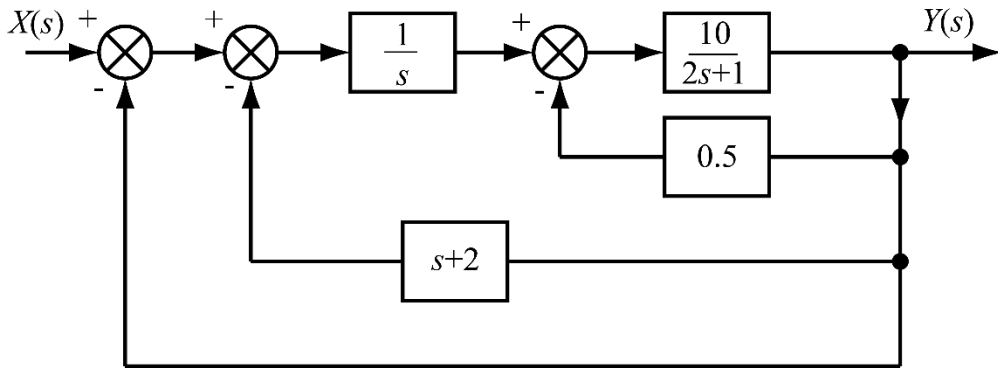
Figure Q4

a. Show whether the above closed-loop system is stable or unstable.   **(8 marks)**

b. Either through knowledge of the response of system types to different inputs or by applying the final value theorem calculate the steady-state error for this system to the following inputs:

    i.   A step input of magnitude 10.
    ii.   A ramp input given by $x(t) = 10t$.
    iii.   A parabolic input given by $x(t) = 10t^2$ **(6 marks)**

c. Calculate an expression for the output time response of the system $y(t)$ when the input is a step input of magnitude 10 if the output response is zero at time $t = 0$.   **(6 marks)**

Question FIVE

    a.   State the effect of introducing feedback on the stability of control systems. **(2 marks)**
    b.
        i.   State the Nyquist criterion.
        ii.   The Nyquist plot of a unity feedback system having open loop transfer function is as shown in Figure Q5a.
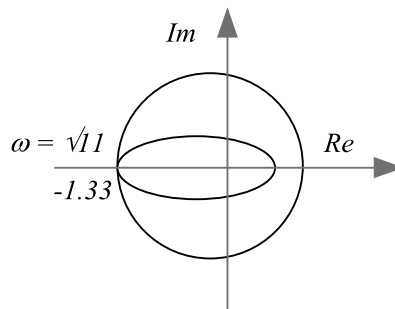

Figure Q5a.

Given the open loop transfer function:

$$G(s) = \frac{K(s+5)(s+3)}{(s-2)(s-4)}$$

When K=1 the system is stable. Determine the range of K for the system to be considered stable.   **(8marks)**

c. Figure Q5b shows a schematic of an armature-controlled d.c. motor which essentially consists of an armature coil in a magnetic field. The armature consists of a resistance, R, and an inductance, L, in series. When current, i, flows through the armature, the coil rotates generating a torque, T, which is proportional to the current, so that $T = K_m i$. Since the armature is rotating in a magnetic field, a voltage, known as the back e.m.f. (e) will be induced in it. The back e.m.f. is proportional to the armature rotation speed, ω, so that $e = K_b \omega$. The motor drives a mechanical load with moment of inertia, I, and with a rotary viscous damping coefficient, c.
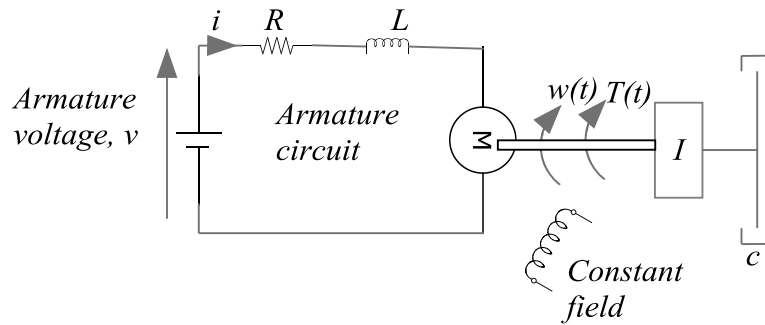

Figure Q5b

Show that the transfer function which relates $\Omega(s)$, the Laplace transform of the armature rotation speed, $\omega(t)$, to $V(s)$, the Laplace transform of the armature voltage, $v(t)$, is given by the following expression: **(10 marks)**

$$G(s) = \frac{\Omega(s)}{V(s)} = \frac{K_m}{(LIs^2 + (RI + cL)s + (Rc + K_b K_m)}$$