TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Engineering and Technology
Department of Mechanical & Automotive Engineering
UNIVERSITY EXAMINATION FOR:
BSc. Mechanical Engineering
EMG 2410 : Control Engineering II
END OF SEMESTER EXAMINATION
SERIES: APRIL 2016
TIME: 2 HOURS
DATE: 15 Apr 2016

Instructions to Candidates

You should have the following for this examination

- *Answer booklet*
- *Non-Programmable scientific calculator*

This paper consists of **FIVE** questions. Attempt question **ONE** and any other **TWO** questions.

Maximum marks for each part of a question are as shown.

Do not write on the question paper.

Question ONE (Compulsory)

a. Most controllers used in industry are the *three term* or *PID controllers*. PID stands for:

- **P**roportional term in the controller.
- **I**ntegral term in the controller.
- **D**erivative term in the controller.

Describe the control action of each term in the controller and define their mathematical model. **(6 marks)**

b. It is found that the behavior of many mechanical systems can be adequately represented as a single-degree-of-freedom system as shown in Figure Q1. The mass of the system is m kg, the linear spring stiffness is k N/m and the damping coefficient is c N/(m/s).

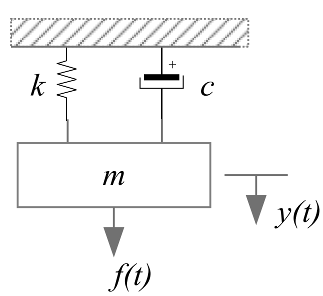

Figure Q1

i. Draw a free body diagram of the system and derive an expression for the relationship between the output displacement, $y(t)$, and the input force, $f(t)$.

ii. Write an expression for the transfer function $G(s)$ which relates the output response $Y(s)$, the Laplace transform of $y(t)$, to the input $F(s)$, the Laplace transform of $f(t)$. Assume that the initial displacement and velocity are both equal to zero.

iii. It is decided that the response of the mechanical system be altered by placing the system transfer function $G(s)$ into a closed-loop negative feedback with a proportional controller having gain K. Calculate the required value of the proportional controller gain, K, which results in the quickest system response without any overshoot. **(12 marks)**

c. Consider the differential equation of a SISO system given by equation (1). Determine the state feedback gain matrix K for the following system such that the closed loop poles are located at -4.8+3.6i, -4.8-3.6i and 4.8.

$$\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = u \qquad (1)$$

(12 marks)

Question TWO

a. In the design of state variable feedback systems, the conditions of controllability and observability may govern the existence of a complete solution to the control system design problem. Define the following terms:

   i. Controllability
   ii. Observability **(4 marks)**

b. Consider a transfer function of a system as shown in Figure Q2

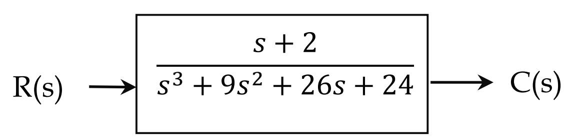

Figure Q2

   i. Find the state equation and output equation for the phase variable representation of the transfer function.

   ii. Check for controllability and observability for the system. **(16 marks)**

Question THREE

a. Let the reference input to the system as shown in Figure Q3 below be a unit ramp and determine the steady-state error for the controller $K(s)$ being:

   i. A PD controller,
   ii. A PI controller **(10 marks)**

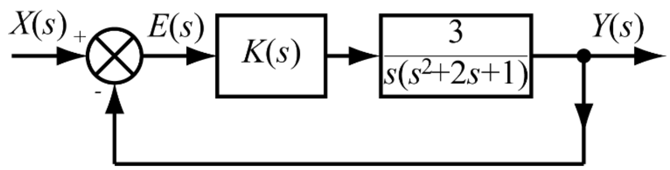

Figure Q3

b. For the general closed-loop system as shown in the previous question (a) but with $G_1(s) = \dfrac{2.5}{s^2 + 0.1s + 0.25}$ calculate the gains K_p and K_d in a PD controller which give a closed-loop damping ratio of 0.5 and a closed-loop natural frequency of 4 rad/s.

(10 marks)

Question FOUR

a. Consider a position control system with low damping where the system output is given in mm and the process input is in volts. Let the system be modelled by the following transfer function:

$$G_1(s) = \frac{K}{s^2 + 0.4s + 1}$$

We wish to control the system behavior and use a PD controller and unity feedback. Assume that the reference signal is a unit step input and the disturbance is given by $d(t) = 0.5$. Determine the effect of introducing the PD controller on the overall performance of the system in terms of stability, reference tracking and disturbance rejection. **(14 marks)**

b. In typical control engineering problems nonlinearity may occur in the dynamics of the plant to be controlled or in the components used to implement the control. State and briefly describe any THREE common nonlinearity that can exist in a system.

(6 marks)

Question FIVE

a. The Figure Q5 shows a typical closed-loop system with the process transfer function $G_1(s)$ and the controller transfer function $K(s)$

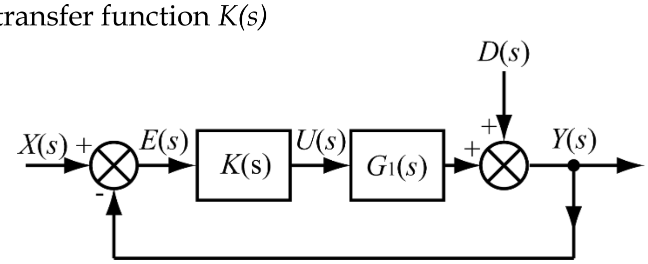

Figure Q5

Determine the closed-loop transfer function, $G(s)$, and the sensitivity (disturbance rejection) transfer function, $S(s)$ for the following systems.

i. $K(s) = K_P + K_d s,\ G_1(s) = \dfrac{32}{s^2 + 2.4s + 16}$

ii. $K(s) = K_P + \frac{K_i}{s}, \; G_1(s) = \frac{12}{s+10}$ **(8 marks)**

b. A speed control loop in a manufacturing plant gave a 0.5m/s change in speed when a 2% reference change was made. The speed measurement was found to be subject to a significant amount of measurement noise. The speed loop time constant was found to be 5 minutes. It is desired to find a controller to ensure no steady-state offset to step reference signals, to achieve a 5% settle time of less than 10 minutes at most and to have as little overshoot as possible. By following the procedure outlined in the Process Model approach, assume a first-order model for the speed loop and design a suitable three-term controller. **(12 marks)**

- Maximum overshoot M_p

$$M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

- Settling time t_s of within 5%

$$t_s = \frac{3}{\xi\omega_n}$$