TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Engineering and Technology
Department of Mechanical & Automotive Engineering
UNIVERSITY EXAMINATION FOR:
BSc. Mechanical Engineering
EMG 2410 : Control Engineering II
END OF SEMESTER EXAMINATION
SERIES: APRIL 2016
TIME: 2 HOURS
DATE: 15 Apr 2016

Instructions to Candidates

You should have the following for this examination

- *Answer booklet*
- *Non-Programmable scientific calculator*

This paper consists of **FIVE** questions. Attempt question **ONE** and any other **TWO** questions.

Maximum marks for each part of a question are as shown.

Do not write on the question paper.

Question ONE (Compulsory)

a. In the design of state variable feedback systems, the conditions of controllability and observability may govern the existence of a complete solution to the control system design problem. Define the following terms:

  i. Controllability
  ii. Observability **(4 marks)**

b. Consider a transfer function of a system as shown in Figure Q1

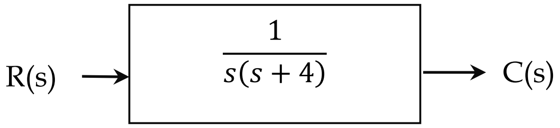

Figure Q1

  i. Find the state equation and output equation for the phase variable representation of the transfer function.

  ii. Evaluate the coefficients of the state feedback gain matrix such that the closed-loop poles have the values -2, -2. **(16 marks)**

c. Consider an RLC network as shown in Figure Q1. Determine the following,

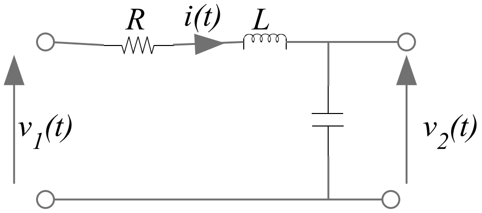

Figure Q1

i. Derive an expression for the relationship between the output voltage, $v_2(t)$, and the input voltage, $v_1(t)$.
ii. Write an expression for the transfer function $G(s)$ which relates the output response $V_2(s)$, the Laplace transform of $v_2(t)$, to the input $V_1(s)$, the Laplace transform of $v_1(t)$. Assume that the initial conditions are equal to zero.
iii. It is decided that the response of the electrical system be altered by placing the system transfer function $G(s)$ into a closed-loop negative feedback with a proportional controller having a gain, K. Calculate the required value of the proportional controller gain, K, which results in the quickest system response without any overshoot. **(10 marks)**

Question TWO

a. Consider the differential equation of a SISO system given by equation (1).

$$\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = u \qquad (1)$$

  i. Check for controllability and observability for the system.
  ii. It is desired to find a controller to achieve a 2% settle time of less than 1 seconds at most and to have as little overshoot as possible (use a damping ratio of 0.8). Determine the coefficients of the state feedback gain matrix in order to accomplish the design requirements. **(20 marks)**
    - Settling time t_s of within 2%

$$t_s = \frac{4}{\xi\omega_n}$$

Question THREE

a. Let the reference input to the system as shown in Figure Q3 below be a unit ramp and determine the steady-state error for the controller $K(s)$ being:
  i. A PD controller,
  ii. A PI controller **(10 marks)**

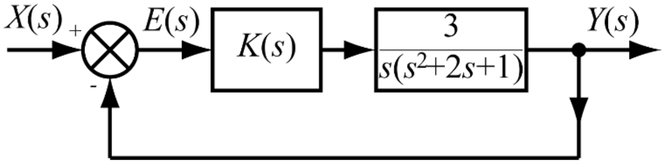

Figure Q3

b. For the general closed-loop system as shown in the previous question (a) but with $G_1(s) = \frac{2.5}{s^2 + 0.1s + 0.25}$ calculate the gains K_p and K_d in a PD controller which give a closed-loop damping ratio of 0.5 and a closed-loop natural frequency of 4 rad/s.

(10 marks)

Question FOUR

a. The plant manager responsible for a chemical plant is unhappy with the performance of one of the processes. When a 10°C reference input step change is applied to the process this only results in a 5°C output temperature change. The time constant of the system is 20 minutes.

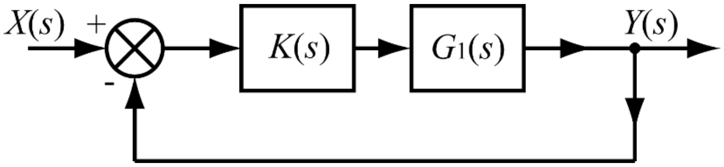

Figure Q4

i. Assuming that the process can be represented by a first-order model, given the above information, specify the system transfer function, $G_1(s)$, relating the output temperature to the input temperature.

ii. Sketch the output response of the process to the 10°C reference input step change, clearly indicating the desired steady-state value, the actual steady-state value and the relevance of the time constant.

iii. Determine the steady-state error of the process.

iv. Initially, the manager is only concerned with the slow system response to the step change and desires that the time-constant of the system should be reduced from 20 minutes to 5 minutes. The closed-loop control system is as shown in Figure Q4 where $G_1(s)$ is the transfer function of the process and $K(s)$ is the controller transfer function. The manager decides to use proportional control to achieve this one design requirement. Show that the required value for the proportional controller gain to satisfy the design requirement is $K_p = 6$.

v. The plant manager subsequently decides to add an integral control component to the proportional component to give perfect reference tracking (zero steady-state error). It is decided that a closed-loop system damping ratio of 1 is desirable. Calculate the value of K_i, the integral controller gain, which satisfies the new design requirement, assuming that the proportional gain is kept at a value of $K_p = 6$.

vi. Comment upon the potential control benefits, if any, of including a derivative block in the controller for the first-order process discussed above. **(20 marks)**

Question FIVE

a. Most controllers used in industry are the *three term* or *PID controllers*. PID stands for:

- **P**roportional term in the controller.
- **I**ntegral term in the controller.

- **D**erivative term in the controller.

Describe the control action of each term in the controller and define their mathematical model. **(6 marks)**

b. It is decided that a controller is required for a process in a chemical works. The open-loop step response is a 5°C output temperature change for a 10°C input change and the open-loop time constant is 10 minutes. The design specifications are:

- No steady-state error in reference tracking mode.
- ±5% settle time of better than 20 minutes but without any overshoot.

i. In order to satisfy the design requirements, which of the PID components should be used?
ii. Write an expression for the forward-path transfer function for the system $G_o(s)$.
iii. State the closed-loop transfer function of the system $G_{CL}(s)$.
iv. Calculate the required value of the controller gains to satisfy the design requirement.
v. Using the Final Value Theorem, or otherwise, what will be the steady-state response of the closed-loop system, with the value of controller gain calculated above, when the reference input is a step of 20°C? **(14 marks)**