



TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED & HEALTH SCIENCES

MATHEMATICS & PHYSICS DEPARTMENT

UNIVERSITY EXAMINATION FOR:

BACHELOR OF TECHNOLOGY IN APPLIED PHYSICS

APS 4212: VECTOR ANALYSIS

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: MAY 2016

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of 4 questions.

Do not write on the question paper. Answer question ONE (compulsory) and any other two questions.

SECTION A (30 MARKS)

QUESTION 1

- (a) (i) Prove that the divergence of the curl of a vector vanishes. [5points]
- (ii) Prove that the gradient operator is a vector operator. [3points]
- (b) (i) For any vector B whose components are given in three dimensional Cartesian coordinates, compute $\nabla \times B$ [6points]
- (ii) Show that $\nabla \cdot (\nabla \times B) = 0$ [4points]
- (c) (i) Write down the expressions for the unit vectors in spherical coordinates, and find their

derivative with respect to each other. [5points]

(ii) Prove that $\mathbf{AXBXC} = \mathbf{B(A.C)} - \mathbf{C(A.B)}$ [5points]

(d) Construct any two 2x2 matrices and show that they do not commute. [2points]

SECTION B

QUESTION 2 (20Points)

(a) (i) Given a 3X3 square matrix A = find its transpose A and compute the product of the matrix and its transpose. [7points]

(ii) Find the inverse of the matrix $A = \begin{bmatrix} 13 \\ 21 \end{bmatrix}$ [5points]

(b) Given the two linear equations $x+3y = 2$ and $2x+y = 3$, use matrix technique to solve for x and y. [8points]

QUESTION 3 (20Points)

(a) (i) Explain what is meant by a vector space. [2points]

(ii) Explain what is meant by a Hilbert space. [3points]

(iii) Explain what is meant by a linear operator. [3points]

(iv) Explain what is meant by linearly dependent set of vectors and a set of linearly independent vector. [2points]

(b) (i) Given the following matrix,

$$A = \begin{bmatrix} (2 + 3i) & \dots & (4 - 5i) \\ 3 & \dots & (4i) \end{bmatrix}$$

compute the Hermitian conjugate A^+ [4points]

(ii) Give an example of a unitary matrix and show that it, actually, is unitary. [6points]

QUESTION 4 (20Points)

(a) If σ is a closed surface which encloses a volume τ , prove that

$$\oint_{\sigma} n d\tau = 0$$

[4points]

(b) Prove that $\iiint_{\tau} \nabla X A d\tau = \oint_{\sigma} n X A d\sigma$

[8points]

(c) Show that $\nabla X \nabla X A = \nabla \nabla \cdot A - \nabla^2 A$

[8points]