TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED &HEALTH SCIENCES

MATHEMATICS & PHYSICS DEPARTMENT

UNIVERSITY EXAMINATION FOR:

BACHELOR OF TECHNOLOGY IN ENVIRONMENTAL PHYSICS & RENEWABLE ENERGY

APS 4303: THEMAL PHYSICS II

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: MAY 2016

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of 4 questions.

Do not write on the question paper. Answer question ONE (compulsory) and any other two questions.

SECTION A (30POINTS)

QUESTION 1

(a) Explain the following terms

(i) Ensemble average [3points]

(ii) Partition function [3points]

(iii) Entropy [3points]

(b) (i) Consider a one-particle system of two states, one of energy 0 and one of energy ε. The particles are in thermal equilibrium with a reservoir at temperature τ. Compute the energy and heat capacity of the system as a function of then temperature τ. [7points]

(ii) If we shift the zero energy and take the energies of the two states as $-\frac{1}{2}\varepsilon$ and $+\frac{1}{2}\varepsilon$, compute the partition function and heat capacity of the system and find what the heat capacity looks like in conventional temperature system. [7points]

(c) Consider a model system with $N_\uparrow$ spins up and $N_\downarrow$ spins down.Let $N = N_\uparrow + N_\downarrow$; the spin excess is $2s = N_\uparrow - N_\downarrow$. The entropy is given by(in Stirling's approximation)

$$\sigma(s) \approx -\left(\frac{1}{2}N + s\right)\log\left(\frac{1}{2} + \frac{s}{N}\right) - \left(\frac{1}{2}N - s\right)\log\left(\frac{1}{2} - \frac{s}{N}\right).$$

In a magnetic field B, what would be the free energy and what is the expression for the minimum energy? [7points]

SECTION B

QUESTION 2

(a) For a particle in a box the energy is given by

$$\varepsilon_n = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2\left(n_x{}^2 + n_y{}^2 + n_z{}^2\right),$$ where the letters have their usual meanings.

(i) Give the expression for the partition function for this system. [4points]

(ii) What is the expression for the partition function if the spacing between adjacent energy is small in comparison with τ. Use the formula,

$$\left(\int_0^\infty dn_x \exp\left(-\alpha^2 n_x{}^2\right)\right)^3 = \frac{\pi^{3/2}}{8\alpha^3}$$ [6points]

(b)The partition function of an ideal of N identical particles is given by

$Z_N = \frac{1}{N\downarrow}\left(n_Q V\right)^N$, where $n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)$ and the letters have their usual meanings.

(i) Determine energy of the gas. [3points]

(ii) Determine the pressure of the gas. [3points]

Iiii) Determine the entropy of the system. [4points]

QUESTION 3

(a) Give a brief description of the Debye model of heat capacity. [10points]

(b) In the Debye model of heat capacity the total energy is given by

$$U = \int_0^{n_D} dnn \frac{\hbar\omega}{\exp(\hbar\omega/\tau) - 1}.$$

(i) Determine the total energy in the low temperature limit. [5points]

(ii) Determine the heat capacity C_V in the low temperature regime. [5points]

QUESTION 4

(a) Define the chemical potential of an ideal gas. [4points]

(b) The free energy of a monatomic gas is given by

$$F = -\tau\left[\log Z_1 - \log N \downarrow\right] \text{where } Z_1 = n_Q V = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{3/2} V$$

From this expression determine the chemical potential. [6points]

(c) The differential of entropy is given by $d\sigma(U,V) = \left(\frac{\partial\sigma}{\partial U}\right)_V dU + \left(\frac{\partial\sigma}{\partial V}\right)_U dV$.

If denote the independent values of dU by $(\delta U)_n$ and dV by $(\delta V)_n$ the entropy change will be zero.

(i) Determine the expression for the pressure in terms of τ, σ, V with U kept constant. [6points]

(ii) From the expression for $d\sigma$ obtain the thermodynamic identities. [4points]