TECHNICAL UNIVERSITY OF MOMBASA
A Centre of Excellence

DEPARTMENT OF MATHEMATICS AND PHYSICS
MAY 2016 SERIES EXAMINATION
UNIT CODE: SMA 2472 UNIT TITLE: OPERATIONAL RESEARCH

SPECIAL/SUPPLEMENTARY EXAMINATION
TIME ALLOWED: 2HOURS

## INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of FIVE questions
Answer question ONE (COMPULSORY) and any other TWO questions
Maximum marks for each part of a question are as shown

## QUESTION ONE (30 MARKS)

a. Define the following terms as used in replacement problem
i. Economic service life
(1 marks)
ii. The challenger
iii. The defender
b. Find the optimal solution for the following transport problem.

| Requirements |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | supply |
| X | 13 | 11 | 15 | 20 | 2 |
| Y | 17 | 14 | 12 | 13 | 6 |
| Z | 18 | 18 | 15 | 12 | 7 |
| demand | 3 | 3 | 4 | 6 |  |
| Where the cost is given in pounds \$ |  |  |  |  | 6 marks) |

c. Posta corporation must make a rectangular steel box whose three dimensions have a sum of 120 cm . what is the maximum volume of the box and what are its dimensions ( 5 marks)
d. Use graphical method to minimize the cost of transporting electrical poles if the linear programme is defined by

$$
\begin{array}{ll}
\operatorname{minimize} \operatorname{cost} C=12 x+20 y \\
\text { subject to } & 4 x+6 y \geq 48 \\
& 3 x+9 y \geq 45 \\
& 14 x+7 y \geq 84 \\
& x, y, \geq 0
\end{array}
$$

e. A firm has developed a new product X . they can either test the market or abandon the project. The details are set out below. Test market cost $\$ 50000$; likely outcomes are favorable ( $\mathrm{p}=0.7$ ) or failure ( $p=0.3$ ). if favorable, they could either abandon or produce it when demand is anticipated to be.

| Low | $\mathrm{p}=0.25$ | loss | $\$ 100000$ |
| :--- | :--- | :--- | :--- |
| Medium | $\mathrm{p}=0.6$ | profit | $\$ 150000$ |
| High | $\mathrm{p}=0.15$ | profit | $\$ 450000$ |

If the test market indicates failure the project would be abadondoned. Abandonment at any stage results in a gain of $\$ 30000$ from the special machining used. Draw the decision tree showing the nodes and probabilities and evaluate the decision tree (8 marks)

## QUESTION TWO (20 MKS)

a. Use simplex method to
maximize $p=4 x+3 y$
subject to $-x+y \leq 4$

$$
\begin{aligned}
& x+2 y \geq 14 \\
& 2 x+y \leq 16
\end{aligned}
$$

$$
x, y \geq 0
$$

b. Free laptops for schools requires Engineers to do the installations. Some four sampled schools are shown with four available Engineers. The distances to the schools are given in kilometers and an assignment to minimize mileage is required. Give the best assignment

| Engineers | Schools where installation is to be done |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | Makau pry <br> school | Chengo pry <br> school | Cheruyot pry. <br> School | Makokha pry <br> school |
|  | Ouma | 25 | 18 | 23 | 14 |
|  | Kariuki | 38 | 15 | 53 | 23 |
|  | Khadija | 15 | 17 | 41 | 30 |
|  | leting | 26 | 28 | 36 | 29 |

c. The following data relate to a given stock item

| Normal usage | 1300 per day |
| :--- | :--- |
| Minimize usage | 900 per day |
| Maximize usage | 2000 per day |
| Lead time | $15-20$ DAYS |
| ROQ | 30000 |

Calculate the various control levels hence state two advantages and one disadvantage of the periodic review system of inventory control

## QUESTION THREE (20 MKS)

a. KPLC is considering investing in one of three investment opportunities $A, B, C$ under uncertain conditions. The pay off matrix for the situation is given as below

| Investment <br> opportunity | $1 \$$ | $2 \$$ | $3 \$$ |
| :--- | :--- | :--- | :--- |
| A | 5000 | 7000 | 3000 |
| B | -2000 | 10000 | 6000 |
| C | 4000 | 4000 | 4000 |

Determine the best investment opportunity using the following criteria
i. Maximin
ii. Maximax
(1 marks)
iii. Minimax
(2 marks)
iv. Hurwicz (alpha=0.3)
b. A wholesaler stocks an item for which demand is uncertain. He wishes to access two reordering policies i.e orders 10 units at a reorder level of 10 , or order i5 units at a reorder level of 15 units, to see which is most economical over a 10 day period.
The following information is available

| Demand per day (units) | probability |
| :---: | :--- |
| 4 | 0.1 |
| 5 | 0.15 |
| 6 | 0.25 |
| 7 | 0.3 |
| 8 | 0.2 |

Carrying costs $\$ 15$ per unit per day. Ordering costs $\$ 50$ per order. Loss of goodwill for each unit out of stock $\$ 30$. Lead time 3 days. Opening stock 17 units. The probability distribution is to be based on the following random numbers.

| 41 | 92 | 5 | 44 | 66 | 7 | 0 | 0 | 14 | 62 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 20 | 7 | 95 | 5 | 79 | 95 | 64 | 26 | 6 | 48 |

Noting that reorder level is level is physical stock plus any replishment orders outstanding
( 6 marks)
c. Pole demand is 5000 units per year. Ordering costs are $\$ 100$ per order and the basic unit price is \$5 carrying costs are 20\% p.a

Discounts are available thus:

| $1200-1399$ | less $10 \%$ |
| :--- | :--- |
| $1400-1499$ | less $15 \%$ |
| 1500 and over | less $20 \%$ |

KPLC wishes to make the most economical order. Find the number of poles to order
(6 marks)

## QUESTION FOUR (20 MARKS)

a. Find the time series analysis for $\alpha=0.1$ and $\alpha=0.5$ by the method of exponential smoothing for the data below

| MON | JAN | FEB | MAR | APR | MAY | JUN | JUL | AUG | SEPT | OCT | NOV | DEC | JAN |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| SALES | 450 | 440 | 460 | 410 | 380 | 400 | 370 | 360 | 410 | 450 | 470 | 490 | 460 |

b. Calculate the cost slopes and the critical path of the following network hence construct a least cost schedule for the network showing all the durations from normal time- normal cost to cash time -crash cost.
(9 marks)

| ACTIVITY | Preceding | time |  | Cost \$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | activity | normal | crash | Normal | crash |
| 1 | - | 5 | 3 | 500 | 620 |
| 2 | - | 4 | 2 | 300 | 390 |


| 3 | 1 | 7 | 6 | 650 | 680 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | 1 | 3 | 2 | 400 | 450 |
| 5 | 2,3 | 5 | 3 | 850 | 1000 |

c. Determine the optimum strategies and the value of the game from the following $2 \times 5$ pay off matrix game for $x$ and $y$
(6 marks)
Y

$$
x\left(\begin{array}{lllll}
6 & 3 & -1 & 0 & -3 \\
3 & 2 & -4 & 2 & -1
\end{array}\right)
$$

## QUESTION FIVE (20 MARKS)

a. Briefly explain the Monte- Carlo techniques
b. MRM Ltd made the following estimates for a component they use;

Annual usage 1125
Ordering costs ksh 50 per order
Carrying costs per year ksh 5 per component.
based on these estimate, an EOQ of 150 and expected total stock costs of 750 were calculated as follows.

$$
E O Q=\frac{\sqrt{2 \times 50 \times 1125}}{5}=150
$$

Expected total stock costs=ordering cost p.a + Holding cost p.a

$$
=\left(\frac{1125}{150}\right) 50+\frac{5 x 150}{2}=750
$$

During the year the EOQ of 150 was used for reordering but the actual usage of components turned to be $20 \%$ higher at 1350 .
i. Calculate the actual total stock costs
(2 marks)
ii. Calculate what the total stock costs would be if a current EOQ was used
(2 marks)
iii. Find out how sensitive total total costs are to errors in the usage estimates.
(2 marks)
c. Draw an activity on node diagram for the following project hence calculate the EST/LST and LFT values for each box
(10 marks)

| Activity | Preceding activity | Duration(days) |
| :---: | :---: | :---: |
| 1 | - | 4 |
| 2 | 1 | 7 |
| 3 | 1 | 5 |
| 4 | 1 | 6 |
| 5 | 2 | 2 |
| 6 | 3 | 3 |
| 7 | 5 | 5 |


| 8 | 2,6 | 11 |
| :---: | :---: | :---: |
| 9 | 7,8 | 7 |
| 10 | 3 | 4 |
| 11 | 4 | 3 |
| 12 | $9,10,11$ | 4 |

## THIS IS THE LAST PRINTED PAGE

