

TECHNICAL UNIVERSITY OF MOMBASA
FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

**UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN
MATHEMATICS AND COMPUTER SCIENCE**

AMA 4410: PARTIAL DIFFERENTIAL EQUATIONS 1

END OF SEMESTER EXAMINATION

SERIES: APRIL 2016

TIME: 2 HOURS

DATE: Pick Date May 2016

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.

Do not write on the question paper. PAPER 1

QUESTION ONE (30 MARKS)

a) Describe the orthogonal trajectories of $y = kx^2, k \neq 0$ [6 Marks]

b) Obtain the general solution to the partial differential equation
 $(y - z)p + (z - x)q = x - y$ [4 Marks]

c) Show that a the partial differential equation arising from
 $z = \frac{1}{2}(a^2 + 2)x^2 + axy + bx + \phi(y + ax)$
can be put in the form $(r + u)(t + v) = s^w$ where u, v, w are integers. [6 Marks]

d) Find the direction cosines of the space curve defined by the parametric equations
 $x = -0.5s^2, y = 0.25s^3, z = 1.5s^2$ through $(-2, -2, 6)$ [6 Marks]

e) Find the complete solution of $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = \sin(3x - y) + 12xy$. [8 Marks]

QUESTION TWO (20 MARKS)

- a) Classify the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + (5 + 2y^2) \frac{\partial^2 z}{\partial x \partial y} + (1 + y^2)(4 + y^2) \frac{\partial^2 z}{\partial y^2} = 0$$

and find its characteristics.

[10 Marks]

- b) Find a complete integral of the equation $p^2 x + q^2 y - z = 0$ using Charpit's method.

[10 Marks]

QUESTION THREE (20 MARKS)

- a) Derive the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$ for a perfectly flexible vibrating string of uniform density

ρ stretched to a uniform density τ between two points $x = 0$ and $x = L$; $c^2 = \frac{\tau}{\rho}$ [8 Marks]

- b) Solve the wave equation in (a) above satisfying the Cauchy conditions

$$u(0, t) = u(L, t) = 0, \quad t \geq 0$$

$$u(x, 0) = f(x), \quad 0 \leq x \leq L$$

$$u_t|_{t=0} = g(x), \quad 0 \leq x \leq L$$

where f and g are given functions

[12 Marks]

QUESTION FOUR (20 MARKS)

- a) Find the General Solution of $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + 5 \frac{\partial^2 z}{\partial y^2} = \sin(3x - y)$

[5 Marks]

- b) Find a partial differential equation arising from the general solution

$$\phi\left(x^6 - y^6, \frac{x^3 + y^3}{z^3}\right) = 0$$

[5 Marks]

- c) Find a complete solution of $p^2 x + q^2 y = z$ using Jacobi method.

[10 Marks]

QUESTION FIVE (20 MARKS)

a) Find the orthogonal trajectories on the conicoid $z(x+y)=4$ of a cone in which it is cut by the system of planes $x-y+z=k$ where k is a parameter. [10 Marks]

b) Find the general integral of the partial differential equation $(2xy-1)p+(z-2x^2)q=2(x-yz)$ and also the particular integral which passes through the line $x=1, y=0$ [10 Marks]