# TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

**UNIVERSITY EXAMINATION FOR:**

BACHALOR OF MATHEMATICS AND COMPUTER SCIENCE

SMA4318: COMPLEX ANALYSIS 1

END OF SEMESTER EXAMINATION

**SERIES:**APRIL2016

**TIME:**2HOURS

**DATE:**May2016

**Instructions to Candidates**

You should have the following for this examination

*-Answer Booklet, examination pass and student ID*

This paper consists of **FIVE** questions. Attemptquestion ONE (Compulsory) and any other TWO questions.

**Do not write on the question paper.**

**Question ONE**

(a) Evaluate $\lim_{z\to\infty} \frac{2z^2+4z+5}{5z+z-5}$ (3mrks)

(b) If a = 6-3i and b = 2 – i ,Evaluate $\frac{a+b}{a}$(3mrks)

(c) Find the conjugate of the complex number $z = \frac{1}{2-3i}$ (3mrks)

(d) Evaluate $\lim_{z\to 3+4i} (\frac{4+z^2}{z})$ (3mrks)

(e) If f(z) = $(z^3 + 4z)^2$ find the derivative of f(z) (4mrks)

(f) show that u (x,y) = $2x - x^3 + 3xy^2$ is harmonic (4mrks)

(g) prove that sec(z) = $\frac{z}{e^{iz}+e^{-iz}}$ (4mrks)

(h) Find all the singular points of the function $f(z) = \frac{i+z^3}{2-3z+z^2}$ (4mrks)

(i) Describe the domain of the function $f(z) = \frac{z}{e^z-1}$(2mrks)

**Question TWO**

(a) Show that the function $f(z)=\frac{-1}{(z-1)\,(z-2)}$ is analytic in the domain $D_1$, : 1z1 < 2 and $D_2$, : 1≤1z1 < 2(6mrks)

(b) Evaluate $\int \frac{1}{4+z^2}$ dz where c: |z-i|=2 (7mrks)

(c) Find the Taylor series for the function $f(Z) = e^{2z}$ (7mrk

**Question THREE**

(a) Solve for $z$ in $e^z = -1$ (5mrks)

(b) Prove that sin (iy)= isinh(y) (5mrks)

(c) Show that log $(1+i)^2$ = 2log (1+i) (5mrks)

(d) Evaluate $(-i)^i$ (5mrks)

**Question FOUR**

(a) If z= 2+5i find$z^{\frac{1}{4}}$ the fourth roots of z (7mrks)

(b) Illustrate the following transformation $f(z) = z^2$ for the line x=1 (7mrks)

c) Show that $f(z) = \frac{z}{\bar{z}}$ is not continuous on the entire Z-plane (3mrks)

d) Evaluate $2i^i$ (3mrks)

**Question FIVE**

(a) Find the derivative of $f(z) = z^2$ by Cauchy theorem (6mrks)

(b) Show that the function $f(z) = 3x + y + (3y - x)i$ is entire (4mrks)

(c) Find the Harmonic conjugate of the function u(x,y) =$e^y$sin(x)
Hence find the function $f(x, y) = u + iv$ (6mrks)

(d) Show that the function in $f(z) = \frac{2z+4}{5z}$ is continuous at z=2 (4mrks)