



TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHALOR OF MATHEMATICS AND COMPUTER SCIENCE

SMA4318: COMPLEX ANALYSIS 1

END OF SEMESTER EXAMINATION

SERIES: APRIL 2016

TIME: 2 HOURS

DATE: May 2016

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.

Do not write on the question paper.

Question ONE

(a) If $z_1 = 3 - 4i$ and $z_2 = 2 + 3i$ evaluate

(i) $\overline{z_1 z_2}$ (4mrks)

(ii) $\arg(z_2)$ (2mrks)

(b) Find the cuberoot of the complex number $z = 4 + 4i$ (4mrks)

(c) Express the equation of the function $f(z) = z^2$ in polar form (4mrks)

(d) Evaluate $\lim_{z \rightarrow \infty} \frac{2z+1}{z+1}$ (4mrks)

(e) If $f(z) = \frac{z^2-1}{2z+1}$, Evaluate the derivative of $f(z)$ (4mrks)

(f) Show that $\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$ (4mrks)

- h) Determine the singular points of the function $f(z) = \frac{2z+1}{(z^2-1)z}$ (4mrks)

Question TWO

- (a) Find the Laurent series of the function $f(z) = e^z$ (8mrks)
- (b) Find the residues of the function $f(z) = \frac{z+1}{z^2(z^2+1)}$ in c where $c: |z| \leq 1$ (7mrks)
- c) If z_0 is any interior to the positive orientation of a simple close contour C and $f(z)=1$ find $\int_c \frac{f(z)}{z-z_0} dz$ (5mrks)

Question THREE

- (a) Solve the equation $e^z = -2$ (5mrks)
- (b) Prove that $\ln z = \ln r + i\theta$ (5mrks)
- (c) Evaluate the differentiation of the function $f(z) = (z^2 + 3z)(\cos z)^2$ (5mrks)
- (d) Evaluate $\int_0^{\pi} e^{it} dt$ (5mrks)

Question FOUR

- (a) Represent graphically the conformal mapping $f(z) = Z^2$ for the line $y=1$ (6 mrks)
- (b) Show that the function $f(z) = \frac{z^2+4z}{z+4}$ is continuous at $z=1$ (3mrks)
- (c) Show that the function $f(z) = \bar{z}$ does not satisfies Cauchy riemann theorem (4mrks)
- (d) Show that the facton $T(x,y) = e^{-y} \sin x$ is harmonic (4mrks)
- (e) Evaluate 2^i (3mrks)

Question FIVE

- (a) Use the Cauchy theorem to find $\frac{dw}{dz}$ given that $w = e^x (\cos y + i \sin y)$ (8mrks)
- (b) Find the harmonic conjugate of a harmonic function $u(x,y) = y^3 - 3x^2 y$ hence find $f(x,y)$ (8mrks)
- (c) If $z = 2 + 4i$ find $|\bar{z}|$ hence find the argument of z (2mrks)