TECHNICAL UNIVERSITY OF MOMBASA

Faculty of applied and Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONIC ENGINEERING

SMA 2480: COMPLEX ANALYSIS

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: 2016

PAPER A

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of 5 questions. Question one is compulsory. Answer any other two questions

Do not write on the question paper.

QUESTION ONE (COMPULSORY)

a) Given that $z_1 = 2+i, \quad z_2 = 3-2i, \quad z_3 = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$ evaluate

i) $|3z_1 - 4z_2|$ (3 marks)

ii) $\left|\dfrac{2z_2 + z_1 - 5 - i}{2z_1 - z_2 + 3 - i}\right|^2$ (4 marks)

b) Derive the Polar form of complex numbers from a point say $A(x, y)$ on the Cartesian plane. (6 marks)

c) Prove that $(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$ where n is a positive integer (3 marks)

d) Obtain the isolated singular points , $\operatorname{Res}\{f(z), a\}$ of the function given by

$$f(z) = \frac{1}{(z-3)(z+1)}$$ (6 marks)

e) Define a Laplace inverse transform (3 marks)

f) Show that if images to two curves under a conformal mapping are orthogonal then the curves are orthogonal (2 marks)

g) Check if the function $z^2 = (x^2 - y^2) + 2xyi$ satisfies the Cauchy Riemann equations (3 marks)

QUESTION TWO

a) State and prove the Cauchy Riemann Equations (10 marks)

b) Evaluate $\oint_c \frac{e^z}{(z+1)^2} dz$ where c is the circle $|z-1| = 3$ (6 marks)

c) Show that $e^{iz} = \cos z + i\sin z$ (4 marks)

QUESTION THREE

a) State and prove the Residue Theorem (10 marks)

b) Using definition of a derivative, obtain the derivative of $w = f(z) = z^3 - 2z$ at

$z = z_0 = 0$ (10 marks)

QUESTION FOUR

a) State and prove the Cauchy Integral Theorem (12 marks)

b) Evaluate $\int_{0}^{4+2i} \bar{z}dz$ along the curve given by $z = t^2 + it$ (8 marks)

QUESTION FIVE

a) Given that $L\{f(x)\} = \frac{s+1}{s^2+s-6}$ obtain $f(x)$ (12 marks)

b) Show that $u = e^{-x}(x\sin y - y\cos y)$ is a harmonic function (8 marks)