

TECHNICAL UNIVERSITY OF MOMBASA
A Centre of Excellence

DEPARTMENT OF MATHEMATICS AND PHYSICS
MAY 2016 SERIES EXAMINATION
UNIT CODE:SMA 2472 UNIT TITLE:OPERATIONAL RESEARCH

## MAIN EXAMINATION

## TIME ALLOWED: 2HOURS

## INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of FIVE questions
Answer question ONE (COMPULSORY) and any other TWO questions
Maximum marks for each part of a question are as shown

## QUESTION ONE (30 MARKS COMPULSORY)

a. Define the following terms as used in network analysis
i. Dummy activity
ii. Event
iii. Network
b. Use simplex method to

$$
\begin{aligned}
& \text { Maximize } \mathrm{p}=2 x+6 y+4 z \\
& \text { subject to } \quad 2 x+5 y+2 z \leq 38 \\
& \qquad 4 x+2 y+3 z \leq 57 \\
& x+3 y+5 z \leq 57
\end{aligned}
$$

$x, y, z \geq 0$
(8 marks)
c. Calculate the 3 and 6 monthly averages of the forecasting data below
(5 marks)

| MON | JAN | FEB | MAR | APR | MAY | JUN | JUL | AUG | SEP | OCT | NOV | DEC |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| SALES | 1200 | 1280 | 1310 | 1270 | 1190 | 1290 | 1410 | 1360 | 1430 | 1280 | 1410 | 1390 |

d. ABC Itd has aggregate demand of 1.2 million units. Each time they place an order, there is an ordering cost of ksh 1000, holding cost is ksh 100 per unit. Determine
i. EOQ
(2 marks)
ii. Number of order to be based on EOQ
iii. Total cost of stocks based on the EOQ
(3 marks)
e. At a power distribution centre in a town there is only one resident Engineer to serve the customers arriving from the rural community. The government is convinced that the situation faced here is a single serve queuing situation with Poisson arrivals and Poisson service. After all, the calling units are the members of the community and the service mechanism is the resident Engineer attending to them. It is known that customers arrive randomly at a rate of 0.2 customers per hour. Each customer requires different amount of time for service. The Engineer reckons, however, that he serves his customers at an average rate of 0.2 customers per hour. Calculate
i. The average number of customers in the queuing system
(1 mark)
ii. The number of customers actually waiting in the queue (2 marks)
iii. The average amount of time a customer spends on a queuing system(1 mark)
iv. The average amount of time a customer spends in the queue (2 marks)
v. The probability of having exactly zero customers in the queue. (2 marks)

## QUESTION TWO (20 MARKS)

a. An Engineer designs a machines power rating for bread production such that type I bread costs ksh 50 to produce and is sold at $x$ shillings while Type II bread costs ksh 60 and is sold at $Y$ shillings. If the company makes (250y-250x) pieces of Type I bread and ( $32000+250 x-500 y$ ) of Type II bread. Determine $x$ and $y$ so that profit is maximized.
(6 marks)
b. KenGen has four sales people who have to visit four clients. The profit records from previous visits are tabled below and it is required to maximize profits by the best assignments (profits are in \$). Work out the best assignment
(6 marks)

| CUSTOMERS | SALES PEOPLE |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | HENRY | GEORGE | STEVE | OSCAR |
|  | 1 | 6 | 12 | 20 | 12 |
|  | 2 | 22 | 18 | 15 | 20 |


|  | 3 | 12 | 16 | 18 | 15 |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | 4 | 16 | 8 | 12 | 20 |

c. A firm produces two products $X$ and $Y$ with contributions of $\$ 8$ and $\$ 10$ per unit respectively. Solve by graphical method and L.P model associated with the data above. (8 marks)

|  | Labour hours | Material A | Material B |
| :--- | :--- | :--- | :--- |
| $X$ | 3 | 4 | 6 |
| $Y$ | 5 | 2 | 8 |
| Total available | 500 | 350 | 800 |

## QUESTION THREE (20 MARKS)

a. A filling station is being planned and it is required to know how many attendants will be needed to maximize earnings. From traffic studies is has been forecasted that the customers will arrive in accordance with the following table.

| Customers | probability |
| :---: | :---: | :---: |
| 0 | 0.72 |
| 1 | 0.24 |
| 2 | 0.03 |
| 3 | 0.01 |

From past experience it has been estimated that service times vary according to the following table.

| Service time <br> in minutes | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| probablity | 0.16 | 0.13 | 0.12 | 0.1 | 0.09 | 0.08 | 0.07 | 0.06 | 0.05 | 0.05 | 0.05 | 0.04 |

If there are more than two customers waiting, in addition to those being serviced, new arrivals drive on and the sales is lost. A petrol pump attendant is paid $\$ 40$ per 8 hour day, and the average contribution per customer is estimated to be \$4. How many attendants are needed?
(6 marks)
b. A lorry costs $\$ 50000$ and it is required to find the least cost point to replace it with a new vehicle. The following data have been estimated. Use the data to approximate the best replacement time
(8 marks)

| year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Resale value | 36000 | 28000 | 22500 | 17500 | 13000 | 10000 | 8000 | 6000 |
| Annual <br> maintenance <br> cost | 2000 | 2600 | 3200 | 4600 | 7000 | 11000 | 13000 | 15000 |

c.
i. A company uses 100000 units per year which cost $\$ 3$ each. Carrying costs are \$250 per order. Find EOQ? (2 marks)
ii. Using (i) above what would be the EOQ if the company made items themselves on a machine with a potential capacity of 600000 units per year ( 2 marks)
iii. Calculate the various control levels given the following information.

| Normal usage | 560 per day |
| :--- | :--- |
| Minimum usage | 240 per day |
| Maximum usage | 710 per day |
| Lead time | $15-20$ days |
| EOQ | 10000 |

## QUESTION FOUR (20 MARKS)

a. Find the optimum strategies and the value of the game from the following pay off matrix concerning two persons game

Player $Y$
Player $\times\left(\begin{array}{ll}1 & 4 \\ 5 & 3\end{array}\right)$
(6 marks)
b. Find the critical path of the following network using the EST/LSTs hence find the floats of the network below.

| Activity | preceeding activity | duration(days) |
| :--- | :--- | :--- |
| 1 | - | 4 |
| 2 | 1 | 7 |
| 3 | 1 | 5 |
| 4 | 1 | 6 |
| 5 | 2 | 2 |
| 6 | 3 | 3 |
| 7 | 5 | 5 |
| 8 | 2,6 | 11 |
| 9 | 7,8 | 7 |
| 10 | 3 | 4 |
| 11 | 4 | 3 |
| 12 | $9,10,11$ | 4 |

Also, if the $\delta$ of the activities on the critical path are $1,2,1.5,3,2.5$ and 3 respectively. Based on these values calculate the probability of achieving a scheduled time of 40 days for the project duration.
(9 marks)
c. A company wishes to optimize profit and realizes electrical appliances Type I yields Revenue R1 defined by $R_{1}=A_{1} Q_{1}-B_{1} Q_{1}{ }^{2}$ while Type II yields revenue $R_{2}$ definued as $R_{2}=A_{2} Q_{2}-$ $B_{2} Q_{2}{ }^{2}$. The total cost of production is defined as.

$$
C=A_{3}+B_{3}\left(Q_{1}+Q_{2}\right)
$$

## QUESTION FIVE (20 MARKS)

a. TUM has a project to make a perimeter wall in one of its campuses with the following activity durations and resources requirements.

| Activity | preceding activity | duration days) | resource requirements(units) |
| :--- | :--- | :--- | :--- |
| A | - | 6 | 3 |
| B | - | 3 | 2 |
| C | - | 2 | 2 |
| D | C | 2 | 1 |
| E | B | 1 | 2 |
| F | D | 1 | 1 |

Assuming no restrictions show the network, critical path and resource requirements on a day by day basis assuming that starts are made on the EST of each activity.State the plan if there are only 6 units of resources?
b. A company with an ageing product range is investigating the launch of a new range. Their business analysts have mapped out several possible scenarios which are given below. Scenario 1
Continue with old range producing projects declining at $10 \%$ p.a on a compounding basis. Last year's profits were $\$ 60000$ from this range
Scenario 2
Introduce a new range without any prior market research. If sales are high, annual profit is put at $\$ 90000$ with a probability which from past data is put at 0.7 . if sales are low, annual profit is put at $\$ 30000$ with a probabity of 0.3
Scenario 3
Introduce a new range with prior market research costing \$ 30000. The market research will indicate whether future sales are likely to be 'good' or 'bad'. If the research indicates 'good', then the management will spend $\$ 35000$ more on capitalequipment and this will increase annual profits to $\$ 100000$ if sales are actually high. If however sales are actually low, annual profits will drop to $\$ 2500$. Should market research indicate 'good' and should management not spend more on promotion then profit levels will be as for scenario 2 above.

If the research indicates bad then the management will scale down their expectations to give annual profits of $\$ 50000$ when sales are actually low. However, if sales do turn out to be high, profits can only rise to $\$ 70000$ because of capacity constraints. Past history of the market research company indicates the following results.

| PREDICTED SALES <br> LEVEL | ACTUAL SALES |  |  |
| :--- | :--- | :--- | :--- |
|  |  | High | Low |
|  | Good | 0.8 | 0.1 |
|  | Bad | 0.2 | 0.9 |

When actual sales were high the market research company had predicted good sales levels $80 \%$ of the time and so on.
Use a time a horizon of 6 years to indicate to the management of the company which
scenario they should adopt
(9 marks)
c. State FOUR circumstances that would force a company management to do replacement analysis in the establishment

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