TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN ELECTRICAL, CIVIL AND MECHANICAL ENGINEERING

SMA 2471 NUMERICAL ANALYSIS 1

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: MAY 2016

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **five** questions. Attempt question **one** and any other two questions.

Do not write on the question paper.

QUESTION ONE

(a) Define an interpolating polynomial. (1 mk)

(b) Evaluate first and second derivatives of $\sqrt{x}$ at x=1.10 given that

x	1.1	1.2	1.3	1.4	1.5
y	-1.62	0.16	2.45	5.39	9.13

(3 mks)

(c) Show that,

$$\left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{Ee^x}{\Delta^2 e^x} = e^x$$

(3 mks)

d) Solve $\frac{dy}{dx} = 1 - y$, y(0) = 0, in the range $0 \le x \le 0.3$ by taking h = 0.1 using the modified Euler's method.

(6 mks)

e) Approximate y(0.6) using Milne's Predictor-Corrector method with h = 0.1 for the equation,

$\frac{dy}{dx} = -2xy$, given that;

x	0.0	0.1	0.2	0.3	0.4
y	1.0000	0.9900	0.9608	0.9139	0.8522

(4 mks)

f) Using Newton's forward interpolating formula, find the missing values in the table of f(x) below:

x	45	50	55	60	65
f(x)	3		2		-2.4

(6 mks)

g) Find a unique quadratic polynomial of degree two or less such that f(0) = 1, f(1) = 3 and f(3) = 55 using the Lagrange interpolation.

(6 mks)

QUESTION TWO

(a) Determine the step size h to be used in the tabulation of $f(x) = \sin x$ in the interval (1,3) so that a linear interpolation is correct to 4 dp.

(7 mks)

(b) A particle moves along a straight line at a time t with it's distance S from a fixed point of the line given by;

$\int \frac{dS}{dt} = t(8-t^3)^{\frac{1}{2}}$. Using the Simpson's $\frac{1}{3}$ rule, calculate the approximate distance travelled by the particle from time t=0.8 to 1.6 using 8 strips correct to 4 decimal paces.

(6 mks)

(c) Using Taylor series method, solve $\frac{dy}{dx} = x^2 - y$, y(0) = 2, at x = 0.1, 0.2, 0.3, and 0.4 correct to 4 decimal places.

(7 mks)

QUESTION THREE

a) Find by the Lagrange's method the function $f(x)$ given the values

x	1	3	4
$f(x)$	6	12	24

Hence find $f(2)$

(7 mks)

b) Evaluate $\int_0^1 e^{-x^2} dx$ using the trapezoidal rule with h = 0.1. (7 mks)

c) By Newton-Raphson method, find the positive root to the equation $2x^2 + 7x - 6 = 0$ correct to 3 significant figures.

(6 mks)

QUESTION FOUR

(a) Use Euler's method to solve

$$\frac{dy}{dx} = \frac{t-y}{2},$$

if y(0) =1 and h = 1, up to n = 2.

(5 mks)

(b) Apply the second order Runge-Kutta method to find y(0.2) if;

$\frac{dy}{dx} = y - x$ where h = 0.1 correct to 4 significant figures.

(7 mks)

(c) Using Gauss' backward interpolation, interpolate the sales of a certain commodity for the year 1976 given that;

Year	1940	1950	1960	1970	1980	1990
Sales (in pounds)	17	20	27	32	36	38

(8 mks)

QUESTION FIVE

a) Integrate $\int_2^3 (x^2 - 2)dx$ by Simpson's one third rule, taking 5 ordinates correct to 4d.p.

(6 mks)

b) Use Romberg's method to evaluate $\int_0^1 \frac{1}{1+x^2}dx$ correct to 4 d.p by taking $h_1 = 0.25$ and $h_2 = 0.125$ correct to 4 d.p. (8 mks)

c) Obtain Picard's second approximate solution of the initial value problem,

$$\frac{dy}{dx} = \frac{x^2}{y^2+1}, \text{ y (0) = 0.}$$ (6 mks)