TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF ENGINEERING & TECHNOLOGY

MECHANICAL ENGINEERING

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE

EMG 2414: Numerical Methods for Engineers

END OF SEMESTER EXAMINATION

SERIES: APRIL 2016

TIME: 2 HOURS

DATE: 2016

Instructions to Candidates

You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of Choose No questions. AttemptChoose instruction.
Do not write on the question paper.

Question ONE

(a) Using Cramer's Rule solve the linear system of equations

$$2x + y = 7$$
$$3x - 4y = 5$$

(5 Marks)

(b) Solve the following system of equations using Gauss elimination method

$$2x_2 + x_3 = -8$$
$$x_1 - 2x_2 - x_3 = 0$$
$$-x_1 + x_2 + 2x_3 = 3$$

(5 Marks)

(c) Let $A = \begin{bmatrix} 7 & 10 \\ 1 & -2 \end{bmatrix}$, find the Eigen values and the corresponding Eigen vectors of A

(5 Marks)

(d) Find $f'(3)$ using the Newton's backward difference formula

x	1	1.5	2	2.5	3
$f(x)$	-1.5	-2.875	-3.5	-2.625	0.5

(5 marks)

(e)   Obtain a divided difference table for the following data

x	-1	0	2	3
$f(x)$	-8	3	1	12

(5 marks)

(f)   Approximate the definite integral

$$\int_0^1 \sqrt{1+x^2}\,dx \quad \text{Using the trapezoidal rule correct to 4 decimal places using } \Delta x = 0.2$$

(5 marks)

Question TWO

(a)   Consider $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

Find i) A^2,

ii) Eigen values and corresponding Eigen vectors of A^2, .   **(7 Marks)**

(b) Use Euler's method to numerically integrate $\dfrac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8.5$ from $x = 0$

to $x = 1$ with a step size of 0.5 the initial condition at $x = 0$, is $y = 1$ **(7 Marks)**

(c)   Using forward difference formula, estimate $f'(x)$ of
$f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$ at $x = 0.5$ using a step size of $h = 0.5$

(6 Marks)

Question THREE

(a) If $y = x^3 - x^2 + x - 1$, calculate the values of y for $x = 0,1,2,3,4,5$ and form the Backward difference table.
(7 Marks)

(b) The table below shows data for $f(x) = 0.2 + 25x^2 - 200x^2 + 675x^3 - 900x^4 + 400x^5$ with unequally spaced values.

x	$f(x)$
0	0.2
0.12	1.309729
0.22	1.305241
0.4	2.456
0.54	3.507297
0.7	2.363
0.8	0.232

Using the trapezoidal rule for unequally spaced values, find $\int_0^{0.8} f(x)dx$ **(7 marks)**

(c) Using Gaussian Elimination, Solve the system of linear equations

$$8x_1 + 5x_2 + 11x_3 = 30$$
$$-x_1 - 4x_2 + 2x_3 = 3 \qquad\qquad \textbf{(6 marks)}$$
$$2x_1 - x_2 + 5x_3 = 12$$

Question FOUR

(a) Show that the second divided difference of $f(x) = \dfrac{1}{x}$, Using the points (a,b,c) is $\dfrac{1}{abc}$

(7 marks)

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(b) (i) State Lagrange's formula of interpolation using unequal intervals.

(2 marks)

(ii) Using Lagrange's interpolation formula, find the value of y corresponding to x=3 from the table below. **(5 marks)**

x	0	1	4
y	1	3	105

(c) Compute $f'(2.0)$ using backward difference table from the following tabular data.

x	1.4	1.6	1.8	2.0
$f(x)$	4.0552	4.9530	6.0496	7.3981

(6 marks)

Question FIVE

(a) (i) State the formula of trapezoidal rule **(2 marks)**

(ii) A curve passes through $\{(0,1),(0.25,0.9412),(0.5,0.8),(0.75,0.64),(1,0.5)\}$

Find $\displaystyle\int_0^1 f(x)dx$ by trapezoidal rule. **(6 marks)**

(iii) How can the accuracy of the trapezoidal rule be increased? **(2 marks)**

(b) Find an approximate value of $\log_e 5$ by approximating $\displaystyle\int_0^5 \dfrac{1}{4x+5}dx$ using Simpson's $\dfrac{1}{3}$ rule of integration using $n = 10$ equal sub- intervals. **(10 marks)**