TECHNICAL UNIVERSITY OF MOMBASA

AMA 5208 TIME SERIES EXAM

INSTRUCTIONS: ANSWER ANY THREE QUESTIONS

QUESTION ONE [20 MARKS]

a) State and explain three reasons why we need to study the trend of a time series [6 marks]

b) The sales of a company in millions of shillings for 1975 to 1979 is given below.

Year(t)	1975	1976	1977	1978	1979
Sales(Xt)	65	92	132	190	275

i. Fit the trend function $X_t = ab^t$ [7 marks]

ii. Compute the trend values and estimate the sales for the year 1980(2 marks)

c) Using the method of 3-selected points fit the logistic curve

$$X_t = \frac{k}{1+e^{a+bt}} \qquad k \neq 0, b < 0$$ [5 marks]

d)

a) Define a positive definite function for $x \in X$ [2marks]

b) Show that the covariance function of the stationery time series {X_t} is positive definite [4 marks]

QUESTION TWO [20 MARKS]

c) How would you distinguish between MA(1) and AR (1) processes [2 marks]

d) Show that the covariance function of the stationery time series {X_t} is positive definite [4 marks]

e) Let {X_t} be a moving average process of order 2 given by $X_t = e_t + ¯ e_{t-2}$ where $\{e_t\}$ is WN (0, 1). Compute the variance of the sample mean $\frac{X_1 + X_2 + X_3 + X_4}{4}$ when $¯ = 0.8$ and when [6marks]

f) Let $= X_t = e^{iªt} = \cos ªt + i \sin ªt$. Let $a_j = \frac{1}{2m+1}$, $\forall j \in [-m, m]$. Show that $Y_t = \sum_{j=-m}^{m} a_j X_{t-j}$ can be expressed as $Y_t = \frac{1}{2m+1} \frac{\sin\left(\frac{2m+1}{2}\right)ª}{\sin\left(\frac{ª}{2}\right)} e^{iªt}$ [8marks]

QUESTION THREE [20 MARKS]

a) Consider a process $Y_t = P_t + e_t$ Where P_t is the permanent component and is e_t the transitory component. Suppose that the e_t are independent with mean 0 and variance $±_e^2$. Further suppose that the e_t satisfies $P_t = P_{t-1} + a_t$ where the a_t are independent , mean 0 variance $±_a^2$ and independent of the e_t. Find the ACF of $W_t = Y_t - Y_{t-1}$. What can you say about the ACF based on its ACF. [5 MARKS]

b) Consider the MA(1) model $Y_t = e_t + 0.6e_{t-1}$. Write this model in the form

$Y_t = e_t + \sum_{j=1}^{\infty} ®_j Y_{t-j}$, that is find an explicit formula for the $®_j$ [5 MARKS]

c) Consider the MA(2) process $Y_t = \mu + e_t - 1.3e_{t-1} + 0.6e_{t-2}$ where e_t are white noise process with $\sigma^2 = 1.8$

   a) Find the variance, autocovariance function and the ACF of Y_t [6 MARKS]

   b) Check whether this process is invertaible [2 marks]

d) Suppose that Y_t is a stationary process. Show that the process of the first differences $\{W_t\}$ given by $W_t = Y_t - Y_{t-1}$ is also a stationary process. [2 marks]

QUESTION FOUR [20 MARKS]

Consider the AR(1) process $X_t = \phi X_{t-1} + e_t$ for $|\phi| < 1$.

a) By successive replacements of the X_t's in the process show that the process converges to an infinite moving average process of white noise. [11marks]

b) Show that the autocorrelation function for this process is given by $r(h) = \phi^h$ [9 marks]