TECHNICAL UNIVERSITY OF MOMBASA
SCHOOL OF APPLIED AND HEALTH SCIENCES
MATHEMATICS AND PHYSICS
UNIVERSITY EXAMINATION FOR:
UNIT: CONTINUUM MECHANICS

UNIT CODE: AMA 4437

## END OF SEMESTER EXAMINATION

## SERIES: MAY SERIES

## TIME: 2HOURS

## Instructions to Candidates

You should have the following for this examination
-Answer Booklet, examination pass and student ID
This paper consists of five questions. Attempt Question one and any other two.
Do not write on the question paper.

## Question ONE

a). Differentiate between Newtonian and Non-Newtonian fluid. (4mks)
b). Define the term:
i. Plasticity
ii. Elasticity (2mks)
iii. Surface forces (Fs) (2mks)
c). Discuss the flow for which $W=Z^{2} \quad(5 \mathrm{mks})$
d). Prove that the contraction of the tensor $A_{q}^{p}$ is a scalar or invariant. (5mks)
e). In an incompressible flow the velocity vector is given by:

$$
\mathrm{V}=\left(6 \mathrm{xt}+y z^{2}\right) \mathbf{i}+\left(3 \mathrm{t}+x y^{2}\right) \mathrm{j}+(\mathrm{xy}-2 \mathrm{xyz}-6 \mathrm{tz}) \mathrm{k}
$$

Verify whether the continuity equation is satisfied. ( 5 mks )
f). Work the terms of the indicated sum

$$
\overline{g_{r s}}=g_{j k} \frac{\partial x^{i}}{\partial x^{-7}} \frac{\partial x^{k}}{\partial x^{-s}} \quad \mathrm{~N}=3 \quad \text { (5mks) }
$$

## Question TWO

a). If $\emptyset=A\left(x^{2}-y^{2}\right)$ represent a possible flow phenomena. Determine the stream function. $\quad(4 \mathrm{mks})$
b). The velocity potential for 2-D flow is

$$
\emptyset=x(2 y-1) \text { at } \mathrm{p}(4,5) \text {. Determine }
$$

i. Velocity (4mks)
ii. Value of the stream function (4mks)
iii. Derive the continuity equation (8mks)

## Question THREE

a). Determine the conjugate metric tensor in cylindrical co-ordinates
b). Show that the contraction of the outer multiplication of the tensor $A^{p}$ and $B_{q}$ is an invariant. ( 6 mks )

$$
\begin{aligned}
& \text { c). Solve the initial value problem } \\
& \begin{array}{l}
\frac{d^{2} v}{d t^{2}}-\frac{2 d v}{d t}-8 \mathrm{y}=0 \\
\mathrm{y}(0)=3 \\
y^{1}(0)=6
\end{array}
\end{aligned}
$$

## Question FOUR

Let T be a second order tensor whose component in the Cartesian System $\left(x_{1}, x_{2}, x_{3}\right)$ are given by:-

$$
(T)_{i j}=T_{i j}=\mathrm{T}=\left|\begin{array}{ccc}
3 & -1 & 0 \\
-1 & 3 & 0 \\
0 & 0 & 1
\end{array}\right|
$$

Given that the transformation matrix between the system is $\left(x_{1}, x_{2}, x_{3}\right)-\left(x_{1}^{1}, x_{2}^{1}, x_{3}^{1}\right)$ is

$$
A=\left|\begin{array}{ccc}
0 & 0 & 1 \\
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\
\frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0
\end{array}\right|
$$

a) Obtain the tensor components $T_{i j}$ in the now co-ordinate system $\left(X_{1}^{1}, x_{2}^{1}, x_{3}^{1}\right) \quad(7 \mathrm{mks})$
b) The stress state tensor at one point is represented by the carding stress tensor components.

$$
\varphi_{i j}=\left|\begin{array}{lll}
1 & a & b \\
a & 1 & c \\
b & c & 1
\end{array}\right|
$$

Where $\mathrm{a}, \mathrm{b}$ and c are constants. Determine the constants $\mathrm{a}, \mathrm{b}, \mathrm{c}$ such that the traction vector on the octahedral is the null vector. ( 7 mks )
c) The carding stress tensor component at the point of a Newtonian fluid, in which the bulk viscosity co-efficient is zero are given by:

$$
\varphi_{i j}=\left|\begin{array}{ccc}
-6 & 2 & -1 \\
2 & -9 & 4 \\
-1 & 4 & -3
\end{array}\right| P_{a}
$$

Obtain the viscor's stress tensor component. ( 6 mks )

## Question FIVE

Under the restriction of small deformation theory the displacement field is given by

$$
\bar{U}=\mathrm{a}\left(x_{1}^{2}--5 x_{2}^{2}\right) \widehat{e_{1}}+\left(2 a x_{1} x_{2}\right) \widehat{e}-(0) \widehat{e_{3}}
$$

a) Obtain the linear strain tensor and linear spin tensor
b) Given the shear modulus $G$ obtain the value of the young modulus E to guarantee the balance at any point of the continuum. ( 10 mks )

