



THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

Faculty of Engineering & Technology

DEPARTMENT OF CIVIL AND BUILDING ENGINEERING

**BRIDGING TO HIGHER DIPLOMA IN BUILDING AND CIVIL
ENGINEERING**

SEMESTER EXAMINATIONS

MAY 2010 SERIES

CALCULUS II

TIME: 2 HOURS

Instructions to Candidates

This paper consists of **TWO** sections Section **I** and **II**.
Section **I** has 30 Marks and Section **II** has 40 Marks.
Attempt **ALL** Questions in Section **I** and **ONLY TWO** Questions from Section **II**.
Calculators and Mathematical tables are allowed.
You should have a graph paper.

SECTION I

Attempt ALL Questions in this Section (30 Marks)

Question ONE

- (a). (i). Civil Engineering students wanted to determine the cross-section area of a river at a construction site. They made the following measurements:

Width = 15.0m

Depth at equal intervals across the river in m:

0, 2.40, 3.60, 4.70, 4.40, 2.80, 0

Use trapezium rule to estimate the cross-section area of a river.

(3 Marks)

- (ii). Use the prismoidal rule to find the volume a frustum of a sphere contained between two parallel planes on opposite sides of the centre, each of radius 9.00cm and each 5.00 from the centre.

(5 Marks)

- (b). Integrate the following functions with respect to x.

(i) $\int \frac{1}{\sqrt{16-x^2}} dx$

(ii) $\int \cos^2 x dx$

(iii) $\int \sin^2 x \cos x dx$

(8 Marks)

- (c). Work out;

$$\int_0^3 (4+p)^2 dp$$

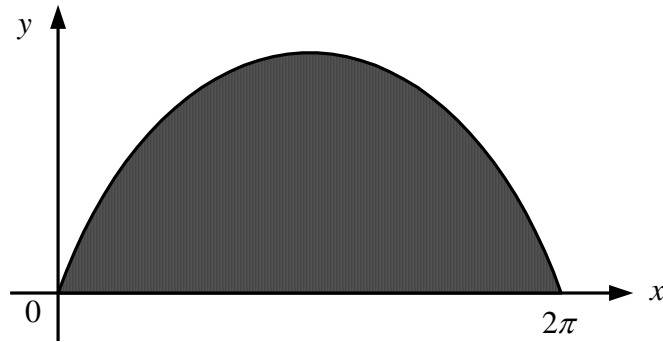
(4 Marks)

- (d). (i) Express, $\frac{2x-1}{(x-1)(2x-3)}$, in partial fractions.

(ii) Hence solve, $\frac{2x-1}{(x-1)(2x-3)} dx$

(6 Marks)

- (e). (i). The curve with equation $y = 3\sin \frac{x}{2}$, $0 \leq x \leq 2\pi$, is shown in Figure I. The finite region enclosed by the curve and the x-axis is shaded.



Find, by integration, the area of the shaded region.

(3 Marks)

(ii). The table below shows three corresponding values of $f(x)$.

x	-1	2	5
$f(x)$	8	26	206

By Simpson's rule with 3 ordinates find an estimate for:

$$\int_{-1}^5 f(x)$$

(3 Marks)

SECTION II

Attempt TWO questions ONLY from this section (40 Marks)

Question TWO

(a). (i). Copy and complete the table below for $y = \frac{1}{2}x^2 - x + 3$ where $0 \leq x \leq 6$.

x	0	1	2	3	4	5	6
y							

(3 Marks)

(ii). Draw the graph of the above function on the grid provided.

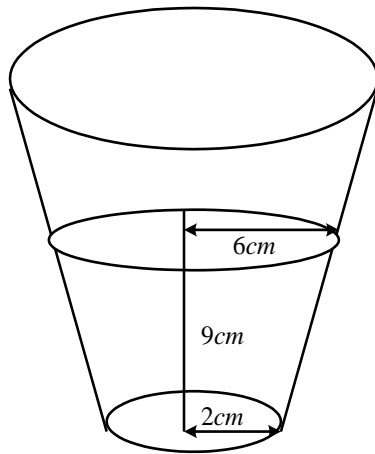
(2 Marks)

(iii). Calculate the mid-ordinates for 6 strips between $x=0$ and $x=6$.

(2 Marks)

- (iv). Use the mid-ordinate rule to calculate the area under the curve. **(2 Marks)**
- (v). Find the area below the curve $y = 1/2x^2 - x + 3$, x -axis
 $x = 0$ and $x=6$ by integration. **(3 Marks)**
- (vi). Calculate percentage error of using mid ordinate rule (in iv)
 assuming that the area calculated in (V) is the actual area. **(2 Marks)**

- (b). A glass in the form of a cone represented by the diagram below. The glass contains water to a height of 9cm. The bottom of the glass is a circle of radius 2cm while the surface of the water is a circle of radius 6cm. **(6 Marks)**



Question THREE

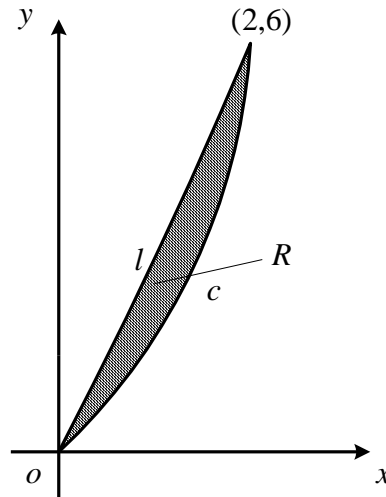
- (a). Evaluate $\int_5^6 \frac{2x^2 - 13x + 13}{(x-4)^3} dx$ **(10 Marks)**
- (b). Find the position of the centroid of the area bounded by the curve
 $y = 4x^2$, $x=1$ and $x=3$. **(10 Marks)**

Question FOUR

- (a). The curve C has equation $y = x\sqrt{x^3 + 1}$ $0 \leq x \leq 2$
- (i). Copy and complete the table below, giving the values of y to 3 decimal places at $x=1$, $x= 1.5$ and $x=2$. **(3 Marks)**

x	0	0.5	1	1.5	2
y	0	0.530			

- (ii). Use the trapezium rule, with all the **y** values from your table, to find an approximation for the value of $\int_0^2 x\sqrt{(x^3 + 1)}$, giving your answer to 3 significant figures. **(4 Marks)**



The figure above shows the curve C with equation $y = x\sqrt{(x^3 + 1)}$ $0 \leq x \leq 2$, and the straight line segment *i*, which joins the origin and the point (2, 6). The finite region R is bounded by C and *i*.

- (iii). Use your answer to part (ii) to find an approximation for the area of R, giving your answer to 3 significant figures. **(3 Marks)**
- (b). (i). Use the identities for $\cos(A + B)$ to prove that $\cos 2A = 1/2(1 + \cos 2A)$ **(4 Marks)**
- (ii). Find the $\int \cos^4 x \sin^2 x dx$ **(6 Marks)**

Question FIVE

- (a). Calculate volume of a frustrum of a sphere of radius 5cm lying between two parallel planes 1cm and 3cm from the centre and on the same side of it. **(6 marks)**
- (b). Area enclosed between the parabolas $x^2 = 2y$ and $y^2 = 16x$ is related about x -axis. Find the volume generated. **(8 marks)**
- (c). Prove by integration that the centroid of a triangle of perpendicular height h and base b lies at a point $h/3$ from the base. **(6 marks)**