TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR THE SECOND SEMESTER IN THE FOURTH YEAR OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

MAY 2016 SERIES EXAMINATION

UNIT CODE: AMA 4423

UNIT TITLE: PARTIAL DIFFERENTIAL EQUATIONS II

TIME ALLOWED: 2HOURS

PAPER B

Instructions to Candidates:

You should have the following for this examination

- *Answer Booklet*
- *Scientific Calculator*

This paper consists of **FIVE** questions and **TWO** sections **A** and **B.**
Answer question **ONE** (**COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages.

SECTION A (COMPULSORY)

Question ONE (30 marks)

a. Consider the following second order partial differential equation:-

$$3u_{xx} + 10xy\, u_{xy} + 3u_{yy} = 0$$

(i) Classify it. (2 marks)

(ii) Reduce to canonical form. (9 marks)

(iii) Find the general solution in terms of arbitrary functions. (2 marks)

b. A string of length L is stretched between points $(0,0)$ and $(L,0)$ on the x axis. At time $t = 0$ it has a shape given by $f(x), \quad 0 \le x \le L$ and it is released from rest.

i. Give the equation of a vibrating string described here (2 marks)

ii. State the boundary and initial conditions associated with this problem (4 marks)

iii. Find the displacement of the string at any latter time t . (11 marks)

SECTION B

Question TWO (20 marks)

a. Solve the Laplace's equation equation $\nabla^2 u = 0$ in two dimension which satisfies the conditions

$$u(0, y) = u(l, y) = u(x, 0) = 0 \quad \text{and}$$

$$u(x, a) = \sin \frac{n\text{®}\, x}{l}$$

by the method of separation of variables. (20 marks)

Question THREE (20 marks)

a. Show that in cylindrical coordinates $r, \text{¯}, z$ defined by the relation $x = r\cos\text{¯}, \; y = r\sin\text{¯}, \quad z = z$, the Laplace's equation $\nabla^2 u = 0$ takes the form $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \text{¯}^2} + \frac{\partial^2 u}{\partial z^2} = 0$ (10 marks)

b. Classify and transform to canonical form $u_{xx} + x^2 u_{yy} = 0$ (10 marks)

Question FOUR (20 marks)

a. Obtain the general solution for $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$ (8 marks)

b. Solve by the method of characteristics $\frac{\partial v}{\partial t} + 3\frac{\partial v}{\partial x} = 0$,

$$v(x,0) = \{ \begin{matrix} \frac{1}{2}x, & 0<x<1 \\ 0, 0therwise \end{matrix}$$

(12 marks)

Question FIVE (20 marks

a. Find the Fourier series expansion of $f(x) = x$ on $(-L, L)$ (8 marks)

b. Solve Laplace’s equation inside a circle of radius a

$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r} \right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \text{¯}^2} = 0$ subject to $u(a, \text{¯}) = f(\text{¯})$ (12 marks)