TECHNICAL UNIVERSITY OF MOMBASA

Faculty of applied and Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE

AMA 4323: ORDINARY DIFFERENTIAL EQUATIONS II

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: 2016

PAPER A

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of 5 questions. Question one is compulsory. Answer any other two questions

Do not write on the question paper.

QUESTION ONE (COMPULSORY)

a) Obtain the general solution to the following homogeneous linear system

$$x_1' = x_2$$
$$x_2' = 3x_2 - 2x_1$$

(5 marks)

b) Find a solution of the initial Value problem $\frac{dy}{dx} = x^2, \quad x_0 = 2, y_0 = 1$ using the uniqueness and existence theorem (5 marks)

c) Reduce the fourth order equation to first order systems

$$\frac{d^4y}{dx^4}-5\frac{d^3y}{dx^3}+7\frac{d^2y}{dx^2}+9\frac{dy}{dx}-6y=e^x$$ (5 marks)

d) Solve $\frac{d^3y}{dx^3}=xe^x$ (5 marks)

e) Solve $x^2y\frac{d^2y}{dx^2}+\left(x\frac{dy}{dx}-y^2\right)=0$ (5 marks)

f) Locate and classify the singular points of the equation

$$(x^4-2x^3+x^2)\frac{d^2y}{dx^2}+2(x-1)\frac{dy}{dx}+x^2y=0$$ (5 marks)

QUESTION TWO

a) Consider a first order vector equation $X'(t)=AX(t)+B(t)$ where A is an $n\times n$ matrix of real numbers, $X(t), B(t)$ are column vectors. Obtain the characteristic polynomial, characteristic equation and Eigen values of matrix A. (12 marks)

b) Solve $zydx=zxdy+y^2dz$ (8 marks)

QUESTION THREE

a) Find the two independent series solutions of the Bessel's equation

$$x^2y''+xy'+(x^2-1)y=0$$ (14 marks)

b) Solve the first order system

$$\frac{dx}{dt}=y,\quad \frac{dy}{dt}=-2x+3y$$ (6 marks)

QUESTION FOUR

a) Solve the system $X' = AX$ where $A = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \end{pmatrix}$ (13 marks)

b) Obtain the roots of the indicial equation of $9x^2 y'' + (x+2)y = 0$ (7 marks)

QUESTION FIVE

a) Solve the system of linear equations

$$x'(t) = 3x(t) - 4y(t)$$
$$y'(t) = 4x(t) - 7y(t)$$

(12 marks)

b) Show the convergence of the initial value problem

$$\frac{dy}{dx} = y; \qquad x_0 = 0, \quad y_0 = 1$$

(8 marks)