TECHNICAL UNIVERSITY OF MOMBASA

Faculty of applied and Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE

AMA 4313: NUMBER THEORY

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: 2016

PAPER B

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of 5 questions. Question one is compulsory. Answer any other two questions

Do not write on the question paper.

SECTION A

Question one

(1)(a) For all integers n. Show that (a,b)=(a-nb,b) (3mks)

(b) Show that a non empty subset A of Z is an ideal if $x, y \in \mathrm{A}$ *then* $x - y \in \mathrm{A}$. (4mks)

(c) Let a,b,c be integers. Show that

(i) (ca,cb)=c(a,b) for every non negative integer c. (4mks)

(ii) If $d = (a,b) \neq 0$ *then* $\left(a/d , b/d \right) = 1$. (2mks)

(d) Let (a,b)=1 and a/bc. Show that a/c. (3mks)

(e) By use of Euclidean Algorithm find (247,91). (4mks)

(f) Let p be aprime number if $p/b_1b_2.......b_n$. Show that p/b_i for some i. (4mks)

(g) Show that there exist infinitely many primes. (4mks)

(h) Show that the equation ax+by=c has integer solutions

(i) If and only if $(a,c)/c$. (4mks)

(ii) If x_0, y_0 is a solution then all solutions are given by

$$x = x_0 + \frac{b}{(a,b)}n,\ y = y_0 - \frac{a}{(a,b)}n,\ n \in Z.$$ (4mks)

(i) Suppose that (a,b)=1. Show that the linear equation ax+by=c has integer solutions For all c (4mks)

SECTION B

Question two

(2)(a) Solve the linear Diophantine equation 247n+91m=39. (5mks)

(b) Solve the equation 6x+10y+15z=5 for integer solutions . (7mks)

(c) Let c be a non zero integer , show that

(i) $f\ ca \cong cb(\text{mod}.m)\ then\ a \cong b(\text{mod}\, m = (c,m))$ (4mks)

(ii) $f\ ca \cong cb(\text{mod}.m)\ and\ (c,m) = 1\ then\ a \cong b(\text{mod}.m)$. (4mks)

Question three ,(20MKS)

(3)(a) State and prove Euler's Theorem (4mks)

(b) Let $f(x) = x^2 + x + 9$. Find the roots of the congruence f(x), 0(mod.63) (6mks)

(c) Show that 2047 is a strong pseudoprime to base 2. (5mks)

(d) By use of wilson's theorem, show that 7 is prime. (5mks)

Question four(20MKS)

(4)(a) Let m be a positive integer. Show that congruences modulo m satisfy

(i) Reflexive property (2mks)

(ii) Symmetric property. (3mks)

(iii) Transitive property. (3mks)

(b)A grocer orders apples and oranges at a total cost of sh.510.If an apple cost him Sh.20 and an orange cost him sh.50. How many of each type of fruit did he order.(6mks)

(c) Find base 2 expansion of 1864. (6mks)

QUESTION FIVE (20MKS) .

(5)(a) Show that (i) $\sum_{j=m}^{n}\left(a_j + b_j\right) = \sum_{j=m}^{n} a_j + \sum_{j=m}^{n} b_j$. (3mks)

(ii) $\sum_{k=-2}^{1} k^3 = -8$ (2mks)

(b) By use of Euclidean algorithm find (34,55). (5mks)

(c) Let b be a positive integer with b>1. Show that every positive integer n can be written uniquely In the form $n = a_k b^k + a_{k-1} b^{k-1} + ... + a_1 b + a_{ij}$, $where\ 0 \le a_j \le b$. (6mks)

(d) Evaluate (i) (105,140,350) . (3mks)

(ii) $\prod_{j=1}^{5} j$. (2mks)