TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN; STATISTICS AND COMPUTER SCIENCE,

MATHEMATICS AND COMPUTER SCIENCE

AMA 4212: VECTOR ANALYSIS

END OF SEMESTER EXAMINATION

SERIES: APRIL 2016

TIME: 2HOURS

DATE: Pick DateSelect MonthPick Year

Instructions to Candidates
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of Choose No questions. AttemptChoose instruction.
Do not write on the question paper.

QUESTION ONE (30 MARKS)

a) Explain each of the following, giving an example of each.

i) A vector (2marks)

ii) A scalar (2marks)

b) Show that addition of vectors is commutative. (4marks)

c) Find the projection of the vector $\hat{i} - 2\hat{j} + \hat{k}$ on $4\hat{i} - 4\hat{j} + 7\hat{k}$. (3marks)

d) Find the divergence and curl of the vector

$\vec{V} = (xyz)\hat{i} + (3x2y)\hat{j} + (xz^2 - y^2 z)\hat{k}$ at the point $(2,-1,1)$ (6marks)

e) A vector field is given by $\vec{A} = (x^2 + xy^2)\hat{i} + (y^2 + x^2 y)\hat{j}$ show that the field is irrotational. (4marks)

f) Given $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\tilde{b} = \hat{i} - \hat{j} + \hat{k}$, $\tilde{c} = \hat{i} - \hat{j} - \hat{k}$. Find the vector $\tilde{a} \times (\tilde{b} \times \tilde{c})$ (5marks)

g) State Green's theorem. (4marks)

QUESTION TWO (20MARKS)

a) Find the angle between the vectors $(2\hat{i} + 6\hat{j} + 3\hat{k})$ and $(12\hat{i} - 4\hat{j} + 3\hat{k})$. (5marks)

b) Use Greens' theorem in a plane to evaluate the integral $\oint_c (2x^2 - y^2)dx + (x^2 + y^2)dy$

Where c is the boundary in the xy plane of the area enclosed by the $x-$axis and the semi-circle $x^2 + y^2 = 1$ in the upper half xy plane. (9marks)

c) A force $\tilde{F} = 3\hat{i} + 2\hat{j} - 4k$ is applied at the point $(1,-1,2)$. Find the moment of the force about the point $(2,-1,3)$ (6marks)

QUESTION THREE (20MARKS)

a). Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\, j$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b.$ (15marks)

b). Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $p(1,2,3)$ in the direction of the line PQ where Q is the point $(5,0,4)$ (5marks)

QUESTION FOUR (20MARKS)

a). Evaluate:

i) $div\left[\frac{\vec{r}}{r^3}\right]$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. (7marks)

ii) $\int_c \vec{F} \bullet \vec{dr}$ where c is the arc of the parabola $y = 2x^2$ from $(0,0)$ to $(1,2)$ and $\vec{F} = 3xy\hat{i} - y^2\hat{j}$ (7 marks)

b). Constant forces $\vec{P} = 2\hat{i} - 5\hat{j} + 6\hat{k}$ and $\vec{Q} = -\hat{i} + 2\hat{j} - \hat{k}$ act on a particle. Determine the work done when the particle is displaced from $A(4,-3,-2)$ to $B(6,1,-3)$

(6marks)

QUESTION FIVE (20 MARKS)

a) If $\vec{a} \times \vec{r} = \vec{b} +$ ª$\vec{a}$ and $\vec{a} \bullet \vec{r} = 3$ where $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = -\hat{i} - 2\hat{j} + \hat{k}$. Find $\vec{r}$ and ª

(9marks)

b) Find the area of a parallelogram whose adjacent sides are $\hat{i} - 2\hat{j} + 3k$ and $2\hat{i} + \hat{j} - 4\hat{k}$

(4marks)

c) Show that the four points $(3,-2,4)$, $B(6,3,1)$, $C(5,7,3)$ and $D(2,2,6)$ are coplanar.(7marks)