INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO

QUESTION ONE (30MARKS)

(a) Integrate the function $f(x) = 3x^2 - 2x + 5$.   (2mks)

(b) (i) Define what is meant by $f : X \longrightarrow Y$ is a continuous function at $x = a$.   (2mks)

    (ii) Investigate continuity of $f(x)$ at $x = -1$ and $x = 1$ where

$$f(x) = \begin{cases} 2 - x, x < -1 \\ x, -1 \le x < 1 \\ 4, x = 1 \\ 4 - x, x > 1 \end{cases} .$$ (4mks)

(c) Find $\dfrac{d^3 y}{dx^3}$ if $y = 2xe^x$ (3mks)

(d) Evaluate the following limits

    (i) $\lim\limits_{x \to 0} \dfrac{\sqrt{x^2 + 16} - 4}{x^2}$.   (3mks)

    (ii) $\lim\limits_{x \to \infty} \dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}$.   (3mks)

(e) Find the area bounded by $x = 4 - y^2$ and the y-axis   (4mks)

(f) Decompose the rational fraction $\dfrac{5x + 2}{(x + 2)(3x - 2)}$ (4mks)

(g) Use first principles to differentiate $f(x) = x^2 + 2x$. Hence find $f'(2)$.   (5mks)

QUESTION TWO (20MKS)

(a) Find the derivatives of

    (i) $g(x) = (x^3 - 3x^2 + 6x + 12)^5$ (3mks)

    (ii) $f(x) = \dfrac{3x - 3}{4x + 9}$ (3mks)

    (iii) $h(x) = \ln(x^6 + 4)$ (2mks)

(b) Use differentials to approximate the value of $\sqrt[6]{1.97}$ (5mks)

(c) Differentiate $f(x) = \sqrt{x - 1}$ and hence find

    (i) the domain of $f'(x)$.

    (ii) the tangent line at $x = 10$.   (7mks)

QUESTION THREE (20MKS)

(a) Differentiate the function $y = \sqrt{x} \sinh^{-1} x^2$ (3mks)

(b) Find the vertical asymptotes of $f(x) = \dfrac{x}{x^2 - x - 2}$. (4mks)

(c) Show that $\dfrac{d}{dx}(\cos ec^{-1}x) = -\dfrac{1}{x\sqrt{x^2 - 1}}$ (4mks)

(d) Find the equation of tangent to the curve $x = 3t^2 + 1$, $y = 2t^3 + 1$ that passes through (4, 3) (5mks)

(e) Find the linearization of $f(x) = \sqrt{x+3}$ at $x = 1$ and use it to approximate $\sqrt{3.8}$. (6mks)

QUESTION FOUR (20MKS)

(a) Integrate by substitution

(i) $\int \sqrt[3]{1 - 3x}\,dx$ (3mks)

(ii) $\int \dfrac{(\ln x)^2}{x}\,dx$ (3mks)

(iii) $\int \dfrac{1}{x^2 + 9}\,dx$ (5mks)

(b) Integrate by parts

(i) $\int x^4 \ln x\,dx$ (4mks)

(ii) $\int x^3 e^x dx$ (5mks)

QUESTION FIVE (20MKS)

(a) Find all intervals where $f(x) = \dfrac{1}{3}x^3 - 3x^2 + 5x - 4$ is increasing or decreasing. (4mks)

(b) Differentiate

(i) $f(x) = e^{x^2} \cosh 4x$. (2mks)

(ii) $y = \dfrac{\sin x}{1 + \cos x}$. (4mks)

(c) A ladder 10m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1m/s, how fast is the top of the ladder sliding the wall when the bottom of the ladder is 6m from the wall? (4mks)

(d) Find the arc length of on $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$ on $\dfrac{1}{2} \le x \le 2$. (6mks)