TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING,

BUILDING & CIVIL ENGINEERING AND MECHANICAL &

AUTOMOTIVE ENGINEERING

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN CIVIL ENGINEERING, MECHANICAL ENGINEERING AND ELECTRICAL ENGINEERING

SMA 2371: PARTIAL DIFFERENTIAL EQUATIONS

END OF SEMESTER EXAMINATION

SERIES:APRIL2016

TIME:2HOURS

DATE: Pick DateMay2016

<u>Instructions to Candidates</u>

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.

Do not write on the question paper. **PAPER 1**

QUESTION ONE (30 MARKS)

a) Describe the orthogonal trajectories of $y = kx^2, k \neq 0$ [6 Marks]

b) Obtain the general solution to the partial differential equation
$(y-z)p + (z-x)q = x - y$ [4 Marks]

c) Show that a the partial differential equation arising from

$$z = \frac{1}{2}(a^2+2)x^2 + axy + bx + ¤(y+ax)$$

can be put in the form $(r+u)(t+v) = s^w$ where u, v, w are integers. [6 Marks]

d) Find the direction cosines of the space curve defined by the parametric equations
$x = -0.5s^2, \quad y = 0.25s^3, \quad z = 1.5s^2$ through $(-2,2,6)$ [6 Marks]

e) Find the complete solution of $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = \sin(3x - y) + 12xy$. [8 Marks]

QUESTION TWO (20 MARKS)

a) Find a partial differential equation arising from the general solution
$¤\left(x^6 - y^6, \frac{x^3 + y^3}{z^3}\right) = 0$ [6 Marks]

b) A long rectangular metal plate has its two long sides and the far end at 0^0 and the base at 100^0. The width of the plate is 10 cm. Find by the method of separation of variables, the steady-state temperature distribution inside the plate. [14 Marks]

QUESTION THREE (20 MARKS)

a) Use Laplace transform method to solve the partial differential equation
$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = u$$

subject to the initial conditions $u(x,0) = e^{-sx}$ and $u(0,s) = 0$

given that $u(x,t)$ is bounded for $t > 0, x > 0$. [10 Marks]

b) An infinite metal plate covering the first quadrant has the edge along the y-axis held at 0^0, and the edge along the x-axis held at
$$u(x,0) = \begin{cases} 100^0, & 0 < x < 1 \\ 0^0, & x > 1 \end{cases}$$

Use the method of separation of variables to find the steady-state temperature distribution as a function of x and y. Assume temperatures of zero as y tends to infinity. [10 Marks]

QUESTION FOUR (20 MARKS)

a) Solve the system

$$y_1^{/} = 4y_1 - 2y_2$$

$$y_2^{/} = y_1 + y_2$$

[14 Marks]

subject to the initial conditions $y_1(0) = 3$ and $y_2(0) = -1$

b) Find the General Solution for $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + 5\frac{\partial^2 z}{\partial y^2} = \sin(3x - y)$ [6 Marks]

QUESTION FIVE (20 MARKS)

a) Find the orthogonal trajectories on the conicoid $z(x + y) = 4$ of a cone in which it is cut by the system of planes $x - y + z = k$ where k is a parameter. [10 Marks]

b) Find the general integral of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ and also the particular integral which passes through the line $x = 1, \quad y = 0$ [10 Marks]

A SHORT TABLE OF LAPLACE TRASFORMS

$f(t)$	$L\{f(t)\}$
1	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+1}$
$\frac{\sin at}{t}$	$\tan^{-1}\frac{a}{s}$ *for* $\operatorname{Re} s > ima$
$\sin at$	$\frac{a}{s^2+a^2}$, *for* $\operatorname{Re} s > ima$
$\cos at$	$\frac{s}{s^2+a^2}$, *for* $\operatorname{Re} s > ima$
$\frac{1}{t}\sin at \cos bt$	$\frac{1}{2}\left(\tan^{-1}\frac{a+b}{s}\right) + \tan^{-1}\left(\frac{a-b}{s}\right)$ *for* $\operatorname{Re} s > 0$
$1 - erf\left(\frac{a}{2\sqrt{t}}\right), a > 0$	$\frac{1}{s}e^{-a\sqrt{s}}$, $\operatorname{Re} s > 0$