TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED &HEALTH SCIENCES

MATHEMATICS & PHYSICS DEPARTMENT

UNIVERSITY EXAMINATION FOR:

BACHELOR OF TECHNOLOGY IN APPLIED PHYSICS

APS 4212: VECTOR ANALYSIS

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: MAY 2016

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of 4 questions.

Do not write on the question paper. Answer question ONE (compulsory) and any other two questions.

SECTION A (30 MARKS)

QUESTION 1

(a) (i) Prove that the divergence of the curl of a vector vanishes. [5points]

(ii) Prove that the gradient operator is a vector operator. [3points]

(b) (i) For any vector B whose components are given in three dimensional Cartesian coordinates, compute $\nabla X B$ [6points]

(ii) Show that $\nabla.(\nabla X B) = 0$ [4points]

(c) (i) Write down the expressions for the unit vectors in spherical coordinates, and find their derivative with respect to each other. [5points]

(ii) Prove that **AXBXC = B(A.C) –C(A.B)**[5points]

(d) Construct any two 2x2 matrices and show that they dot commute. [2points]

SECTION B

QUESTION 2 (20Points)

(a) (i) Given a 3X3 square matrix A = find its transpose A and compute the product of the matrix and its transpose. [7points]

(ii) Find the inverse of the matrix A = $\begin{bmatrix} 13 \\ 21 \end{bmatrix}$ [5points]

(b) Given the two linear equations

x+3y = 2 and 2x+y = 3, use matrix technique to solve for x and y. [8points]

QUESTION 3 (20Points)

(a) (i) Explain what is meant by a vector space. [2points]

(ii) Explain what is meant by a Hilbert space. [3points]

(iii) Explain what is meant by a linear operator. [3points]

(iv) Explain what is meant by linearly dependent set of vectors and a set of linearly independent vector. [2points]

(b) (i) Given the following matrix,

$$A = \begin{bmatrix} (2+3i)........(4-5i) \\ 3..................(4i) \end{bmatrix}$$

compute the Hermitian conjugate A^{+} [4points]

(ii) Give an example of a unitary matrix and show that it, actually, is unitary. [6points]

QUESTION 4 (20Points)

(a) If ± is a closed surface which encloses a volume ^{2}, prove that

$$\oiint_{\pm} nd = 0$$ [4points]

(b) Prove that $\iiint_{2} \nabla X A d^{2} = \oiint_{\pm} n X A d\pm$ [8points]

(c) Show that $\nabla X \nabla X A = \nabla \nabla . A - \nabla^{2} A$ [8points]