TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN; STATISTICS AND COMPUTER SCIENCE, MATHEMATICS AND COMPUTER SCIENCE, INFORMATION TECHNOLOGY

SMA2101/AMA 4103: CALCULUS I

END OF SEMESTER EXAMINATION

SERIES: APRIL2016

TIME: 2HOURS

DATE: Pick DateSelect MonthPick Year

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of Choose No questions. AttemptChoose instruction.

Do not write on the question paper.

Question ONE (COMPULSORY)

(a)i) Let $f(x) = x^2$, $g(x) = x^2 - 4x + x - 8$, evaluate $g(f(x))$ (3 marks)

ii). Given that $f(x) = x^3$, calculate $\frac{f(1+h) - f(1)}{h}$ and simplify (3 marks)

(b). Compute the following limits:

i). $\lim_{x\to 3} \frac{x^2 - 9}{x - 3}$ (2 marks)

ii). $\lim_{x\to 0} \frac{\sqrt{x+4} - 2}{x}$ (3 marks) (c). Determine whether the following function is continuous at $x = 3$

$$f(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3} & ,x \leq 3 \\ 4 & x = 3 \end{cases}$$ (4 marks)

(d). Differentiate i). $\sqrt{1 - x^2}$ (3 marks)

ii). $\frac{3\sin 2x}{2x^3}$ (3 marks)

(e). $s = t^3 - 2t^2 + 3t$, find $\left.\frac{ds}{dt}\right|_{t=-2}$ and $\left.\frac{d^2s}{dt^2}\right|_{t=-2}$ (4 marks)

(f). Find the equation of the normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point (1,2) (5 marks)

<u>Question TWO (20 marks)</u>

(a) Find the derivative of the function $y = x^2 - \frac{3}{2}x$ with respect to x from first principles. (5marks)

(b) Show that $y = e^{2x}$ satisfies the second order linear ordinary differential equation.

$$(2x + 1)\frac{d^2y}{dx^2} - 4(x + 1)\frac{dy}{dx} + 4y = 0$$ (5marks)

(c) A canvas wind shelter for the beach has a back, 2 square sides and a top. Suppose that 96sq. metres of canvas are to be used, find the dimension of the shelter for which the space inside the shelter (i.e. the volume) will be maximized. (8marks)

(d) Differentiate $\left[e^{-3x}\left(1 + e^{6x}\right)\right]^{12}$ (2marks)

<u>Question THREE (20 marks)</u>

(a) Find the derivatives of the following functions:

i) $f(x) = \frac{1}{1n\left(x^4 + 5\right)}$ (3marks)

ii) $y = \frac{\sin x - \cos x}{\sin x + \cos x}$ (4marks)

iii) $y = \tan^{-1} \dfrac{2x}{1-x^2}$ (6marks)

(b) A stone is thrown upwards, so that its height S metres above the ground t seconds is

$s(t) = 76t^2 + 128t + 5$. Find:-

i) Its velocity and acceleration after 2 seconds. (4 marks)

ii) When the stone is 117m above the ground? (3marks)

<u>Question FOUR (20 marks)</u>

(a) Show that $\dfrac{d}{dx}\left(\sin^{-1} x\right) = \dfrac{1}{\sqrt{1-x^2}}$ (4marks)

(b) Given $xy + x - 2y = 5$ find $\dfrac{dy}{dx}$ (4marks)

(c) Evaluate the following limits:-

i) $\lim_{x \to 0} \dfrac{x}{\sin 7x}$ (4marks)

ii) $\lim_{x \to 0} \dfrac{\tan x}{x}$ (4marks)

iii) $\lim_{x \to \infty} \dfrac{4x^2 - 21x + 6}{3x^3 + x^2 - 9}$ (4marks)

<u>Question FIVE (20 marks)</u>

(a) Use differentials and the function $y = \sqrt[3]{x}$ to approximate $y = \sqrt[3]{126}$ (6marks)

(b) The function y of x is given by the parametric equations.

$$\left.\begin{array}{l} x = a\cos t \\ y = a\sin t \end{array}\right\} 0 \le t \le ®$$

Find the derivative $\dfrac{dy}{dx}$ for $t = \dfrac{®}{4}$ (4marks)

(c) Find the turning points on the curve $y = \sin x + \cos x$ (6marks)

(d) A rectangular area is formed using a wire 36cm long. Find the length and breadth of the rectangle if it is to enclose maximum possible area. (4marks)