## MATHEMATICS

TIME: 2 HOURS

## Instructions to Candidates

This paper consists of TWO Sections: Section I and II.
Section I: has 30 marks and Section II has 40 marks.
Attempt ALL Question in Section I and ONLY TWO Questions from Section II. Calculators and mathematical tables allowed.

## SECTION I

## Question ONE

Differentiate following functions:
(i). $\quad \sin \left(3 x^{2}+2\right)$
(ii). $e^{4 t}$
(4 Marks)

## Question TWO

Solve:
$\frac{d y}{d x}=x y-y$
(3 Marks)

## Question THREE

The distribution shown represents marks awarded to students after an assessment.

| Marks | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 5 | 8 | 7 | 12 | 28 | 20 | 10 | 10 |

Determine:
(a). First quartile
(b). Second quartile
(c). Third quartile
(6 Marks)

## Question FOUR

Integrate the following functions with respect to x .
(a). $\frac{1}{\sqrt{\left(9-x^{2}\right)}}$
(b). $(x-2)^{2}$
(6 Marks)

## Question FIVE

(a). Determine the area of a parallelogram whose sides are given by the vectors.
$A=3 i-3 j+2 k$
$B=-2 i-3 j+2 k$
(5 Marks)
(b). Determine a positive constant $\alpha$ such that the angle between the vectors. $u=\alpha(i+j)$ and $v=i+\alpha j$ is $\frac{\pi}{6}$ radius.
(6 Marks)

## SECTION II

## Attempt TWO Questions ONLY from this Section (40 Marks)

## Question SIX

A company monitored the number of days(x) of business trips taken by executives of the company and the corresponding claims ( $£ \mathrm{y}$ ) they submitted to cover the total expenditure of these trips.

A random sample of 10 trips gave the following results.

| $\mathbf{X}$ (days) | 10 | 3 | 8 | 17 | 5 | 9 | 14 | 16 | 21 | 13 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{Y} £$ | 116 | 39 | 85 | 159 | 61 | 94 | 143 | 178 | 225 | 134 |

(a). Plot these data on a scatter diagram.
(4 Marks)
(b). Find an equation of the regression time of y and x in the form of $y=a+b x$.
( 6 Marks)
(c). Find the expected expenditure of a trip lasting 11 days.(2 Marks)

A machine hire company kept records of the ages $x$ months, and the maintenance costs, $£ y$, of machine. The following table summarizes the data for a random sample of 10 machines.

| Machine | A | B | C | $\mathbf{D}$ | $\mathbf{E}$ | F | G | H | $\mathbf{I}$ | $\mathbf{J}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Age x | 62 | 12 | 34 | 81 | 51 | 14 | 45 | 74 | 24 | 89 |
| Maintenance <br> costs y | 111 | 25 | 41 | 181 | 64 | 21 | 51 | 145 | 43 | 241 |

Calculate to (3 decimal places) the product-moment correlation coefficient.
(8 Marks)

## Question SEVEN

(a). (i). A construction company investment on machinery was Kshs.150,000/- in the first year and Kshs.250,000/- each for the following 15 years.
Determine the company's total investment in machinery after 10 years.
(3 Marks)
(ii). Determine an approximate value of $\sqrt{10}$ by substituting $x=\frac{1}{9}$ in the binomial expansion of $(1+x)^{1 / 2}$ upto the term in $x^{3}$.
(8 Marks)
(b). The fourth term of a geometric series is 10 and the seventh term of the series is 80 . For this series, find:
(a). the common ratio
(4 Marks)
(b). the first term
(3 Marks)
(c). the sum of the first 20 terms, giving your answer to the nearest whole number.
(2 Marks)

## Question EIGHT

(a). The figure below shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is $x$ metres.


The capacity of the tank is $100 \mathrm{~m}^{3}$.
(i) Show that the area $\mathrm{Am}^{2}$ of the sheet metal used to make the tank is given by:

$$
\begin{equation*}
A=\frac{300}{x}+2 x^{2} \tag{4Marks}
\end{equation*}
$$

(ii) Use calculus to find the value of x which A is stationary.
(4 Marks)
(iii) Prove that this value of $x$ gives a minimum value of $A$.
(2 Marks)
(iv) Calculate the minimum area of sheet metal needed to make the tank
(2 Marks)
(b). (i). Express $\frac{2 x-1}{(x-1)(2 x-3)}$ in partial fractions.
(3 Marks)
(ii). Given that $x \geq 2$, find the general solution of the differential equation

$$
(2 x-3)(x-1) \frac{d y}{d x}=(2 x-1) y
$$

(5 Marks)

