# THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE 

## Faculty of Engineering and Technology

DEPARTMENT OF BUILDING AND CIVIL ENGINEERING

HIGHER DIPLOMA IN BUILDING \& CIVIL ENGINEERING
(HD A08)

## MATHEMATICS

FINAL EXAMINATION
SERIES: APRIL/MAY 2010
TIME : 2 HOURS

## Instructions to Candidates:

You should have the following for this examination

- Answer booklet
- Mathematical table / scientific calculate

Answer any FIVE of the following EIGHT questions.
All questions carry equal marks.

## Question ONE

a) Given that $Z=\left(A t^{n}+B t^{-n}\right) \operatorname{Sin}(n \theta+\omega$ where $A, B, n$, and $\omega$ are constants show that

$$
\begin{equation*}
\frac{\partial^{2} Z}{\partial t^{2}}+\frac{1}{t} \frac{\partial Z}{\partial t}+\frac{1}{t^{2}} \frac{\partial^{2} Z}{\partial \theta^{2}}=0 \tag{8marks}
\end{equation*}
$$

b) Show that $f(x, y)=x^{2} y^{2}(1-x-y)$ has a maximum at $\mathrm{x}=\mathrm{y}=2 / 5$.

## Question TWO

a) Solve for $a$ in the equation:

$$
\left|\begin{array}{ccc}
a & 4 & 2  \tag{8marks}\\
a^{2} & 4 & 4 \\
a^{3} & 4 & 8
\end{array}\right|=3
$$

b) Given the matrix $A=\left[\begin{array}{lll}2 & 3 & 1 \\ 1 & 2 & 1 \\ 2 & 2 & 1\end{array}\right]$
i) Evaluate $P=A^{2}+3 A$
ii) Find $\mathrm{P}^{-1}$ and hence solve the simultaneous equation
$15 x+23 y+9 z=3$
$9 x+15 y+7 z=-5$
$14 \mathrm{x}+18 \mathrm{y}+8 \mathrm{z}=0$
(12marks)

## Question THREE

a) i. State the necessary and sufficient condition for an equation Mdx + Ndy $=0$ to be exact
ii. Show that the differential equation
$(2 x+3 \operatorname{Cos} y) d x+(2 y-3 x \operatorname{Sin} y) d y=0$ is exact and hence solve the differential equation given that when $x=0, y=\frac{\pi}{2}$
b) Solve the differential equation completely $\frac{d^{2} y}{d x^{2}}-4 \frac{d y}{d x}+3 y=x^{2}+e^{2 x}$ given that $x=0, y=-1, \dot{y}=0$

## Question FOUR

a) In an experiment, the length of 100 white mice are measured to the nearest 0.1 cm and the frequency tabulated as follows:

| Length | $20-24$ | $25-29$ | $30-34$ | $35-39$ | $40-44$ | $45-49$ | $50-54$ | $55-59$ | $60-64$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Freq. | 1 | 4 | 15 | 20 | 25 | 26 | 7 | 1 | 1 |

Find:-
i) The modal and the media class.
ii) Upper quartiles and lower quartile hence interquartile range.
iii) Mean
iv) Median
v) Mode
vi) Standard deviation and skewness
vii) Coefficient of variation
b. A machine manufacturing screws is known to produce 5\% defective to a random sample of 15 screws. What is the probability that there are:
i) Exactly three defective
ii) Not more than three defective.

## Question FIVE

a) Determine the modules and the argument of

$$
\begin{equation*}
Z=\frac{1}{12+5 j} \tag{3marks}
\end{equation*}
$$

b) Solve the equation $Z^{3}=\frac{4-2 j}{j}$ and express yours roots in the form $a+j b$.
c) Given $Z=2-j$ is a root to the polynomial
$3 Z^{3}=14 Z^{2}+23 Z+\lambda=0$
Determine:
i) The value of $\lambda$
ii) The other two roots
d) Use De-Moirres theorem to prove that

$$
\begin{equation*}
\tan 3 \theta=\frac{3 \tan \theta-\tan ^{3} \theta}{1-3 \tan ^{2} \theta} \tag{3marks}
\end{equation*}
$$

## Question SIX

Given that $f(x)=\left\{\begin{array}{c}\frac{c^{2}}{2} e^{-c x} \\ 0_{1}\end{array} \quad x \geq 0\right.$ elsewhere is a probability density
function, determine:
i) The value of $c$.
ii) The expected value of $x$
iii) The standard deviation $\sigma$
iv) The probability that $0 \leq \mathrm{x} \leq 3$ correct to 4 d.p.

## Question SEVEN

a) i. Derive Newton-Raphson iterative formulae.
ii. Use the Newton - Raphson Formulae to determine the cube-root of 123 correct to fire decimal places.
b) Table 1 shows measurements taken by a surveyor where one value was wrongly recorded.

| $x$ | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{f}(\mathrm{x})$ | 5 | 5.64 | 6.36 | 7.16 | 8.4 | 9. | 10.04 |

i) Use finite difference to correct the wrongly recorded value
ii) Determine the function $\mathrm{f}(\mathrm{x})$ by use of Newton - Gregory interpolation formula
(11marks)
iii) Find f(1.48)

## Question EIGHT

a) Evaluate the following interval:

$$
\begin{equation*}
\int \frac{d \theta}{5+4 \operatorname{Cos} \theta} \tag{5marks}
\end{equation*}
$$

b) State the Cauchy linear equation, hence solve the equation

$$
\begin{equation*}
x^{2} \frac{d^{2} y}{d x^{2}}+5 x \frac{d y}{d x}+3 y=\left(1+\frac{1}{x}\right) \text { In } x \text { using the substitution } x=e^{z} . \tag{15marks}
\end{equation*}
$$

