TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

HIGHER DIPLOMA IN BUILDING & CIVIL ENGINEERING

AMA 3201: ORDINARY DIFFERENTIAL EQUATIONS

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2013
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

Question One (Compulsory)

a) State the necessary condition for differential equation to be considered linear. **(2 marks)**

Hence with reason state whether:

$$3\frac{d^3y}{dx^3}+3y\frac{d^2y}{dx^2}+4\frac{dy}{dx}+y=e^{2x}$$

is linear **(2 marks)**

b) Show that the differential equation $\left(2x^3+3y\right)dx+\left(3x+y-1\right)dy=0$ is exact hence find its general solution **(8 marks)**

c) Solve the linear fractional equation $\frac{dy}{dx}=\frac{x+2y-3}{x+y-3}$ to obtain the general solution **(8 marks)**

Question Two

a) Distinguish between an ordinary differential equation and partial differential equation. **(2 marks)**

b) Confirm that $y=Ax^2e^x$ is the general solution to $\frac{dy}{dx}=\frac{y\left(x+2\right)}{x}$ **(5 marks)**

Hence find the particular solution given y = 1 when x = 0 **(10 marks)**

c) Determine the general solution of $x\frac{dy}{dx}=2-4x^3$ **(3 marks)**

d) Solve $\frac{dy}{d\theta}=\sec\theta+y\tan\theta$ given the boundary conditions y = 1 when $\theta=0$ **(5 marks)**

Question Three

a) By separation of variables solve $x+1\frac{dy}{dx}=x\left(y^2+1\right)$ **(4 marks)**

b) Solve $\frac{1}{x}\frac{dy}{dx}+4y=2$ given the boundary conditions x = 0 and y = 4 **(6 marks)**

c) Solve $\left(x+2y\right)\left(dx-dy\right)=dx+dy$ **(10 marks)**

Question Four

a) An object moves with simple harmonic motion on the x axis initially its located at distance 46m away from the origin when t = 0 and velocity v = 15m/s and decelerating at 100m/s^2 directed towards the origin 0. Find the equation of position at any time t.

(Hint F = -kx and for S.H.M, $\mu = \dfrac{Kx}{M}$ **(6 marks)**

b) Obtain differential equation associated with the equation $y = C_1 e^{-x} + C_2 e^{2x}$ **(8 marks)**

c) Find the particular solution for initial value problem $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - 3y = 0$ given y(0) = 0 y' and (0) = -4 **(4 marks)**

Question Five

a) A particle falls in a vertical line under gravity and the force of air resistance to its motion is proportional to its velocity. Show that the velocity cannot exceed a particular limit. (Hint – the equation of motion is given by $\dfrac{dv}{dt} = g - Kv;$ where g is the gravity and K proportionality constants respectively. **(5 marks)**

b) Show that the equation $\left(2xy + 3y^2\right)dx - \left(2xy + x^2\right)dy = 0$ is homogenous. Hence solve the equation. **(8 marks)**

c) Show that the equation $\left(4x + 2y^2\right)dx + 2xy\ dy = 0$ is not exact but has an integrating factor (I.F) of the form x_n. Hence solve the equation **(7 marks)**