<u>**Instructions to Candidates:**</u>
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FOUR** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **TWO** printed pages

Question One (Compulsory)

a) Show that $y = e^{2x}$ is a solution of the differential equation $y'' + y' - 6y = 0$ **(3 marks)**

b) The differential equations a family of circles is given by $\dfrac{dy}{dx} + \dfrac{x}{y} = 0$:
   (i) Find the equations to the family of circles **(2 marks)**
   (ii) If one of the family passes through (1, 2) find its equation **(2 marks)**

c) Show that the given differential equation as homogeneous and hence solve it
$$\left(x^3 + y^3\right)dx - 3xy^2 dy = 0$$

(7 marks)

$$\frac{dx}{dt} + 2x = 4e^{3t}$$

d) Using Laplace Transform, solve   at t = 0 if x = 1 **(5 marks)**

$$(3x^2 + 4xy)dy + (2x^2 + 2y)dy = 0$$

e) Show that the equation: is exact and hence solve it.

(5 marks)

f) Using the D-operators find the complete solution of:

$$\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{2x}$$

(6 marks)

Question Two

$$ydx - xdy = 0$$

a) Show that   is not exact **(2 marks)**

$$\frac{1}{x^2}$$

b) Show that   is an integrating factor for the above equation **(3 marks)**

$$ydx - xdy = 0$$

c) Solve the equation   using integrating factor method **(4 marks)**

$$\frac{dy}{dx} + \frac{1}{x-1}y = xy^{1/3}$$

d) Solve the Bernoulli's equation **(6 marks)**

e) Find the singular points of the differential equation and determine whether they are regular as irregular singular points **(5 marks)**

$$(1-x)y'' \frac{(1-x)y'}{x^2} + \frac{y}{x^2} = 0$$

Question Three

$$E = 100\sin 40t$$

a) A circuit has a series electromotive force given by   a resistor of 10Ω and an inductor of

$$L\frac{di}{dt} + Ri = E$$

0.5 Henry. If the initial current i = 0 at t = 0 and the basic differential   , solve the equation and find the current i at t > 0

(8 marks)

$$(D^2 + D - 2)y = 2x - 40\cos 2x$$

b) Solve the equation   by the method of undetermined coefficient.

(12 marks)

Question Four

a) Find two linearly independent series solutions of the differential equation $y''+xy'+y=0$ about the point x = 1 **(12 marks)**

b) By reduction of order method, solve $y''-y=ex$ **(8 marks)**

Question Five

a) Solve $y''-3y'+2y=\dfrac{1}{1-e^{-x}}$ by variation of parameters method **(10 marks)**

b) Solve $yy''=(y')^2$ by reducing the order by substitution $y'=p$ and $y''=p\dfrac{dp}{dy}$ **(5 marks)**