TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF TECHNOLOGY INFORMATION TECHNOLOGY

SMA 2271: PARTIAL DIFFERENTIAL EQUATIONS (PDE)

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2013
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Consider the differential equation

$$xu_x + yu_y = 0$$

obtain the characteristic equations and the characteristic curves. **(2 marks)**

b) Determine the arc length function for $\vec{r}(t) = < 2t, 3\sin(2t), 3\cos(3t) >$ on the interval $0 \le t \le 2\pi$ **(3 marks)**

c) Solve the following partial differential equation by separation of variables method:

$$3\frac{\partial u}{dx}+2\frac{\partial u}{\partial y}=;\ u(x,0)=4e^{-x}$$

(5 marks)

$$\frac{dx}{x+y}=\frac{dy}{x+y}=\frac{dz}{-(x+y+2z)}$$

d) Find the integral curve of the equation **(4 marks)**

e) Verify that the equation $zydx-zxdy-y^2dz=0$ is integrable and hence find the solution to it.

(5 marks)

f) Find the equation of the tangent plane to the surface $z=xy$ at the point P (2, 3, 6) **(5 marks)**

g) Find the general solution of the following first order differential equation:

$$(x^2+1)y'+6xy=x$$

(4 marks)

h) Give an implicit definition of a surface in R^3 **(2 marks)**

Question Two

a) Find the orthogonal trajectories on the conicoid $z(x+y)=4$ of a conic in which it cut by the system of planes $x-y+z=k$ where k is a parameter **(9 marks)**

b) Solve the differential equation:

$$x(y^2-a^2)dx+y(x^2-z^2)-z(y^2-a^2)dz=0$$

(6 marks)

c) Find the integral surface of the equation $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ which contains the straight lines $x+y=0$ and z = 1 **(5 marks)**

Question Three

a) Prove that a curve C given by $\vec{r}(\theta)=a\left(\cos\theta\,\vec{i}+\sin\theta\,\vec{j}\right);\ a\neq 0,\ -\infty\theta<\infty$ is a regular curve but not a smooth curve **(6 marks)**

b) Find the integral surfaces to the equations:

$$\frac{dx}{x+z}=\frac{dy}{y}=\frac{dz}{z+y^2}$$

(7 marks)

c) Find $u(x,t)$ satisfying $u(x,0) = f(x)$ by the method of separation variables $u(0,t) = u(1,t) = 0$

(7 marks)

Question Four

a) Solve $\frac{\partial u}{dt} = c\frac{\partial^2 u}{\partial x^2}$ using Laplace transforms given the boundary conditions:

$t < 0, \quad \mu(x,t) = 0 \quad t > 0 \quad u(o,t) = 4$

$x \to \infty \quad u(\infty,t) = 0 \quad t = 0 \quad u(x,0) = 0$

(16 marks)

b) Form a semi linear Lagrange's equation from $\varphi(x^2 + y^2 + z^2 \quad xyz) = 0$ **(4 marks)**

Question Five

a) Solve the following first order partial differential equation:

$$\frac{\partial u}{\partial x} + u = e^{-x}$$

(6 marks)

b) Find the general solution of $aux - yuy + y^2u = y^2$ **(6 marks)**

c) Solve the following equation:

$$yz(y+z)dx + xz(x+z)dy + xy(x+y)dz = 0$$

(8 marks)