THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

Faculty of Engineering & Technology

DEPARTMENT OF CIVIL AND BUILDING ENGINEERING

BRIDGING TO HIGHER DIPLOMA IN BUILDING AND CIVIL ENGINEERING (HD/B/09A)

END OF COURSE EXAMINATIONS

APRIL/MAY 2010 SERIES

CALCULUS II

TIME: 2 HOURS

Instructions to Candidates

This paper consists of two sections;
Section I and II
Section I has 30 marks and Section II has 40 marks.
Attempt **ALL** Questions in Section **I** and **ONLY TWO** Questions from Section II.
Calculators and mathematical tables are allowed.

SECTION I

Question ONE

(a). (i). Civil engineering students wanted to determine the cross-section area of a river at a construction site. They made the following measurements:

width = 15.0m
Depth at equal intervals across the river in m:

0, 2.40, 3.60, 4.70, 4.40, 2.80, 0
Use trapezium rule to estimate the cross-section area of a river.

(3 Marks)

(ii). Use the prismoidal rule to find the volume a frustum of a sphere contained between two parallel planes on opposite sides of the centre, each of radius 9.00cm and each. 5.00cm from the centre.

(5 Marks)

(b). Integrate the following functions with respect to x.

(i) $\int \frac{1}{\sqrt{(16-x^2)}} dx$

(ii) $\int \cos^2 x \ dx$

(iii) $\int \sin^2 x \cos x \ dx$

(6 Marks)

(c). Work out;

$\int_0^3 (4-p)^2 dp$

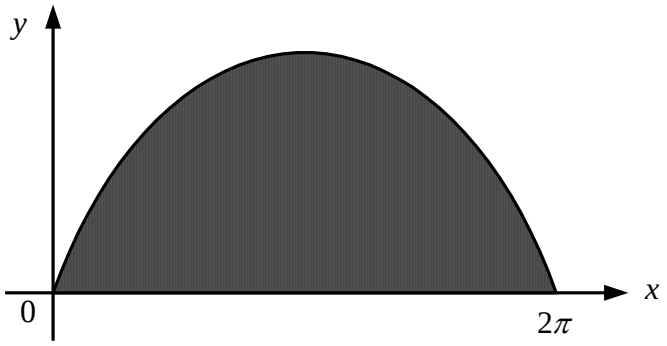
(4 Marks)

(d). (i). Express, $\frac{2x-1}{(x-1)(2x-3)}$, in partial fractions.

(ii). Hence solve, $\int \frac{2x-1}{(x-1)(2x-3)} \ dx$

(6 Marks)

(e). (i). The curve with equation $y = 3\sin\frac{x}{2}, 0 \le x \le 2\pi$, is shown in Figure I. The finite region enclosed by the curve and the x-axis is shaded.



Find, by integration, the area of the shaded region.

(3 Marks)

(ii). The table below shows three corresponding values of f(x).

x	-1	2	5
f(x)	8	26	206

By Simpson's rule with 3 ordinates find an estimate for:

$\int_{-1}^{5} f(x)dx$ **(3 Marks)**

SECTION II

Attempt TWO questions ONLY from this section (40 Marks)

Question TWO

(a). (i). Copy and complete the table below for $y = \frac{1}{2}x^2 - x + 3$ where $0 \le x \le 6$.

x	0	1	2	3	4	5	6
y							

(3 Marks)

(ii). Draw the graph of the above function on the grid provided.

(2 Marks)

(iii). Calculate the mid-ordinates for 6 strips between x=0 and x=6.

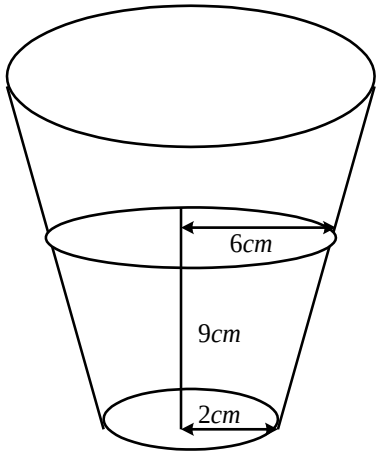
(2 Marks)

(iv). Use the mid-ordinate rule to calculate the area under the curve. **(2 Marks)**

(v). Find the areas below the curve $y = \frac{1}{2}x^2 - x + 3,\ x - axis$ x = 0 and x=6 by integration. **(3 Marks)**

(vi). Calculate percentage error of using mid ordinate rule (in iv) assuming that the area calculated in (e) is the actual area. **(2 Marks)**

(b). A glass in the form of a cone represented by the diagram below. The glass contains water to a height of 9cm. The bottom of the glass contains water to height of 9cm. The bottom of the glass contains water to height of 9cm. The bottom of the glass is a circle of radius 2cm while the surface of the water is a circle of radius 6cm.



Question THREE

(a). Evaluate $\int_5^6 \frac{2x^2 - 13x + 13}{(x-4)^3} dx$ **(10 Marks)**

(b). Find the position of the centroid of the area bounded by the curve $y = 4x^2,\ x = 1\ \ and\ \ x = 3.$ **(10 Marks)**

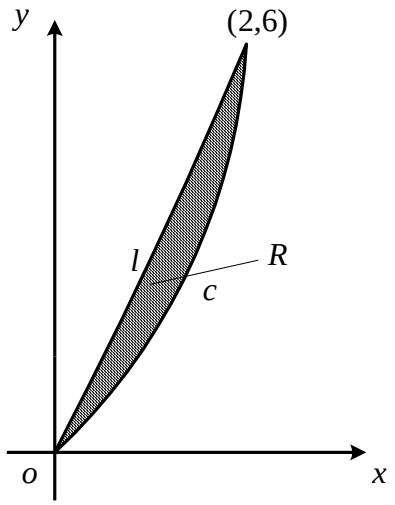
Question FOUR

(a). The curve C has equation $y = x\sqrt{(x^3 + 1)}$ $0 \le \times \le 2$

(i). Copy and complete the table below, giving the values of y to 3 decimal places at x=1, x= 1.5 and x=2. **(3 Marks)**

x	0	0.5	1	1.5	2
y	0	0.530			

(ii). Use the trapezium rule, with all the y values from your table, to find an approximation for the value of $\int_0^2 x\sqrt{(x^3 + 1)}dx$ giving your answer to 3 significant figures. **(4 Marks)**



The figure above shows the curve C with equation $y = x\sqrt{(x^3 + 1)}$ $0 \le \times \le 2$, and the straight line segment /, which joins the origin and the point (2, 6). The finite region R is bounded by C and /.

(iii). Use your answer to part (ii) to find an approximation for the area of R, giving your answer to 3 significant figures. **(3 Marks)**

(b). (i). Use the identities for $\cos(A + B)$ to prove that $\cos 2A = \frac{1}{2}(1 + \cos 2A)$ **(4 Marks)**

(ii). Find the $\int \cos^4 x \sin^2 x dx$ **(6 Marks)**

Question FIVE

(a). Calculate volume of a frustrum of a sphere of radius 5cm lying between two parallel planes 1cm and 3cm from the centre and on the same side of it. **(6 Marks)**

(b). Area enclosed between the paraboles $x^2 = 2y$ and $y^2 = 16x$ is related about x-axis. Find the volume generated. **(8 Marks)**

(c). Prove by integration that the centroid of a triangle of perpendicular height h and base b lies at a point $h/3$ from the base. **(6 Marks)**