



# THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of JKUAT)

*Faculty of Applied & Health Sciences*

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN CIVIL/ELECTRICAL &  
ELECTRONICS/MECHANICAL/BUILDING & CONSTRUCTION & BACHELOR OF  
ENGINEERING IN ELECTRICAL & ELECTRONICS

SMA 2172: CALCULUS I  
AMA 4103: CALCULUS I

SPECIAL/SUPPLEMENTARY EXAMINATION  
SERIES: FEBRUARY/MARCH 2012  
TIME: 2HOURS

### Instructions to Candidates:

You should have the following for this examination

- Answer booklet

This paper consists of **FIVE** questions

Answer Question **ONE (Compulsory)** from **SECTION A** and any other **TWO** questions from **SECTION B**

Maximum marks for each part of a question are clearly shown

This paper consists of **THREE** printed pages

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### SECTION A (Compulsory)

#### QUESTION ONE (30 MARKS)

a) Define the following terms:-

- (i) A function (2 marks)
- (ii) A limit of a function (2 marks)

$$f(x) = x^2 + x - 1 \quad h(x) = x^2 - x$$

b) If \_\_\_\_\_ and \_\_\_\_\_

Find:

- (i)  $f \circ h(x)$  (2 marks)

- (ii)  $h \circ f(x)$  (2 marks)

$$\frac{dy}{dx}$$

c) Find for the following functions:-

$$y = \sin^3 2x$$

(i)

$$x = 2t + 3, y = t^2 - 1 \text{ at } t = 6$$

(ii)

(2 marks)

$$\lim_{x \rightarrow \infty} \frac{(2x-3)(3x+5)(4x-6)}{3x^3 + x - 1}$$

d) Examine the limit of

(4 marks)

e) Which of the following two functions grows faster  $y = 3^x$  or  $y = 2^x$  as  $x \rightarrow \infty$ . Explain (4 marks)

$$f(x) = 2x + 3$$

f) Find the derivative of

by the first principle

(3 marks)

$$\lim_{x \rightarrow 3} (2x + 1 = 7)$$

g) Prove that the limit

(4 marks)

$$\int \frac{x dx}{x^2 + a^2}$$

h) Evaluate the following integral

(3 marks)

## **SECTION B (Attempt any TWO questions)**

### **QUESTION TWO (20 MARKS)**

a) Define the continuity of a function at a point  $x = x_0$  (4 marks)

b) Find the tangent to the curve  $x^2 + xy + y = 7$  at the point (1, 2) (4 marks)

c) Newton's law of universal gravitation states that the force between any particles leaving masses  $M_1$  kg and  $M_2$  kg, separated by a distance  $r$ (m) is an attraction acting along the line joining having the same value for all pairs of particles. Two asteroids are approaching each other. The first has a mass of 1000kg and the second a mass 3000kg.

(i) What is the gravitational force between the asteroids when they are 10km apart? (2 marks)

(ii) How is their force changing at that distance? Explain (3 marks)

$$\lim_{x \rightarrow \infty} \left( \frac{x-1}{x+1} \right)^x$$

d) Evaluate (3 marks)

$$f(x) = \frac{x}{x^2 - 4x + 3}$$

e) Find the vertical asymptotes of (4 marks)

**QUESTION THREE (20 MARKS)**

a) Find the area bounded on the right by the line  $y = x - 2$ , on the left by the parabola  $x = y^2$  and below by the x-axis (6 marks)

b) A bacteria population is growing at a rate equal to 10% of its population each day. Its initial size is 10,000 organisms. How many bacteria are present after 10 days and after 30 days.

c) Find  $f'(x)$  of the following (3 marks)

$$f(x) = (x^3 + x^2 + 1)(x^{19} + 16)$$

d) (i) Find the derivative of  $h(x) = x^2 \sin x$  (2 marks)

$$g(x) = \frac{x-2}{2x-5}, \quad g^{-1}(x)$$

(ii) Given find (3 marks)

**QUESTION FOUR (20 MARKS)**

a) Find:

(i)  $\lim_{x \rightarrow 0} \left( \frac{1 - \cos 2x}{x \sin x} \right)$  (3 marks)

(ii)  $\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 2x}$  (3 marks)

b) (i) Find the derivatives of  $y = (x^2 + 1)^3(x^3 - 1)^2$  and  $y = \frac{4x^3 + 6}{x - 1}$  (6 marks)

(ii) If  $y = t^2 - 1$  and  $x = 2t + 3$

$$\frac{dy}{dx}$$

Find (3 marks)

c) A square sheet tin, which has the measurement of one centimeter on each side, is used to make an open top box by cutting a small square of tin from each corner and bending up the sides. How large a square should be cut from each corner for the box to have as large a volume as possible (5 marks)

**QUESTION FIVE (20 MARKS)**

a) Evaluate :

(i)  $\int x e^x dx$  (3 marks)

(ii)  $\int_0^{\frac{\pi}{2}} x \cos x dx$  (3 marks)

b) A hard-boiled egg at 98°C is put in a sink of 18°C water to cool. After 5 minutes, the egg's temperature is found to be 38°C. Assuming that the water has not warmed appreciably, how much longer will it take for the egg to reach 20°C? (8 marks)

c) Investigate continuity of  $f(x)$  at  $x = -1$  and  $x = 1$  where;

$$f(x) = \begin{cases} 2 - x, & x < -1 \\ x, & -1 \leq x < 1 \\ 4, & x = 1 \\ 4 - x, & x > 1 \end{cases}$$

(6 marks)