THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of Jkuat)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

INSTITUTIONAL BASED PROGRAMME

UNIVERSITY EXAMINATIONS FOR DEGREE IN
BACHELOR OF ENGINEERING IN ELECTRICAL & ELECTRONIC/
MECHANICAL/CIVIL ENGINEERING
YR II SEM 1

SMA 2270: CALCULUS III

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: FEBRUARY/MARCH 2012
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer booklet*

This paper consists of **FIVE** questions
Answer question **ONE (COMPULSORY**) and any other two questions
This paper consist of **THREE** printed pages

Question One (30 marks)

a) Evaluate the following limits

$$\lim_{x\to -2} \frac{x^3+2x^2-1}{5-3x}$$

(i) (4 marks)

$$\lim_{x\to \infty} x\sin\frac{1}{x}$$

(ii) (4 marks)

b) State the Rolle's Theorem (4 marks)

c) Verify the validity of Rolle's Theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ (5 marks)

d) Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$ (6 marks)

e) Find the value of $\frac{df}{dt}$ at $t = \frac{\pi}{2}$ if $f(x, y) = xy$ and $x = \cos t \quad y = \sin t$, (7 marks)

Question Two (20 marks)

a) The semicircle $y = \sqrt{r^2} - x^2$ is revolved about the x-axis to generate a sphere. Find the volume of the sphere (6 marks)

b) Find the value of $\frac{dy}{dx}$ at $t = -2$ for the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2} \quad x = 2 + t,\ y = -t - 1, z = t$ (5 marks)

c) Evaluate the following integral

(i) $\int_0^2 \int_{x^2}^{2x} (4x + 2)\, dydx$ (4 marks)

(ii) $\int_1^e \int_1^e \int_1^e \frac{1}{xyz}\, dxdydz$ (5 marks)

Question Three (20 marks)

a) State Mean Value Theorem (4 marks)

b) Find the value of c $\in$ (a, b) guaranteed by Mean Value Theorem for $f(x) = x^2 + 2x - 1, a = 0, b = 1$ (6 marks)

c) Find the Maclaurin series for $f(x) = In(x + 1)$ up to the term in x^5 (10 marks)

Question Four (20 marks)

a) The arc of the parabola $y = x^2$ from (1, 1) to (2, 4) is rotated about the y-axis. Find the area of the resulting surface (10 marks)

$$\lim_{(x,y)\to(1,1)} \frac{x^2-2xy+y^2}{x-y},\ x \neq y$$

b) Evaluate (4 marks)

$$f(x)=\cos x \ \ at \ \ a=2\pi$$

c) Find the Taylor series generated by (6 marks)

Question Five (20 marks)

$z = x + f(u)$ $u = xy,$ $x\frac{\partial z}{\partial x} - y\frac{dz}{dy} = x$

a) If where show that (5 marks)

b) Find the volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis and $y = x$ $x = 1$ $z = f(x,y) = 3 - x - y$ the lines and and whose top lies in the plane (6 marks)

$$f(x)\begin{cases} \frac{xy^2}{x^2+y^4},\ (x,y) \neq 0 \\ 0,\ (x,y)=0 \end{cases}$$

c) Show that the function is continuous at every point except at the origin. (5 marks)

$$\frac{\partial^3 f}{\partial x \partial y \partial z} \ if \ \ f(x,y,z) = xy + yz + zx$$

d) Find (4 marks)