THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of JKUAT)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONICS ENGINEERING/MECHANICAL ENGINEERING/BUILDING & CIVIL ENGINEERING

SMA 2271: ORDINARY DIFFERENTIAL EQUATIONS

SPECIAL/SUPPLIMENTARY EXAMINATION
SERIES: FEBRUARY/MARCH 2012
TIME: 2HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer booklet*

This paper consists of **FIVE** questions
Answer Question **ONE** (**Compulsory)** from **SECTION A** and any other **TWO** questions from **SECTION B**
Maximum marks for each part of a question are clearly shown
This paper consists of **THREE** printed pages

SECTION A (Compulsory)

QUESTION ONE (30 MARKS)

a) State the necessary conditions for a differential equation to be considered linear (2marks)

Hence with reasons state whether the equation $3\frac{d^3y}{dx^3}+3y\frac{d^2y}{dx^2}+4\frac{dy}{dx}+y=e^{2x}$ is linear (2 mark)

b) Differentiate between general and particular solution of a differential equation. Hence show that the function $f(x)=\left(x^3+c\right)e^{-3x}$ where c is an arbitrary constant is a solution of the differential equation $\frac{dy}{dx}+3y=3x^2e^{-3x}$ (4 marks)

c) Given the differential equation $\left(Ax^2 + Bxy + Cy^2 \right) dx + \left(Dx^2 + Exy + Fy^2 \right) dy = 0$, show that the equation is exact if $B = 2D$ and $E = 2C$ (3 marks)

d) Prove that the transformation $u = y^{1-n}$ reduces the equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$ to a linear equation in u and $x.$ Hence solve the equation $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}$ (7 marks)

e) Given the differential equation $a_o(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0$ explain each of the following statements in relation to a power series solution.

(i) Ordinary point of the equation (1 mark)
(ii) Regular singular point of the equation (1 mark)
(iii) Hence using Taylor's series expansion, find a power series solution of the equation $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0$ $y(1) = 0,\ y^{/}(1) = 4$ (6 marks)

f) Obtain a general solution of the equation $\left(x^2 + 2y^2 \right) dx - 2xydy = 0$ (4 marks)

SECTION B (Attempt any TWO questions)

QUESTION TWO (20 MARKS)

a) By separation of variables solve $y\tan x\frac{dy}{dx} = \left(4 + y^2\right)\sec^2 x$. (4 marks)

b) Solve the linear fractional equation $\frac{dy}{dx} = \frac{x + y - 3}{x - y - 1}$ to obtain the general solution. (6 marks)

c) Find the power series solution of the differential equation using the method of frobenius

$$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x - 5)y = 0$$

(10 marks)

QUESTION THREE (20 MARKS)

a) Solve $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$ (7 marks)

b) An object moves with simple harmonic motion on the x axis. Initially it is located at a distance 46 m away from the origin when t=0 and has velocity v=15 m/s and decelerating at $100m/s^2$ directed towards the origin O. find the equation of the position at any time t. (6 marks)

c) Find the particular solution for the initial value problem $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$ if $y(0) = 0$, $y'(0) = -4$. (7marks)

QUESTION FOUR (20 MARKS)

a) Solve $\frac{dy}{dx} + y\cot x = \cos x$ to obtain the particular solution given that at $x = \frac{\pi}{2}$ When $y = \frac{5}{2}$. (5 marks)

b) Obtain a general solution of the equation $(x^2 - xy + y^2)dx - xydy = 0$. (7 marks)

c) An electric circuit consists of an inductance of 0.1 Henry a resistance of 20 ohms and a condenser of capacitance 25 microfarads. Find the charge q and the current i at any time t, given that the initial conditions are q =0.05 coulombs and $i = \frac{dq}{dt} = 0$ when t =0 if $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E(t)$. (8 marks)

QUESTION FIVE (20 MARKS)

a) Show that $(y^3 + 2x)dx + (3xy^2 + 1)dy = 0$ is exact and then find its general solution. (5 marks)

b) The initial temperature of a body is $53^0 c$ and after 5 minutes its temperature is $45^0 c$, from Newton's law of cooling it is known that the rate of cooling of a body is proportional to the temperature difference between the body and its surrounding room temperature. Use this to predict the temperature of the body after a further 5 minutes given that the room temperature was constant at 21^0C. (7 marks)

c) Using laplace transform solve $\frac{dx}{dt} + 2x = 4e^{3t}$ given that at $t = 0,\ x = 1$. (8 marks)

THE END