THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of JKUAT)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

SMA 2371: PARTIAL DIFFERENTIAL EQUATIONS

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: FEBRUARY/MARCH 2012
TIME: 2HOURS

<u>Instructions to Candidates:</u>
You should have the following for this examination
- *Answer booklet*

This paper consists of **FIVE** questions
Answer Question **ONE** (**Compulsory**) from **SECTION A** and any other **TWO** questions from **SECTION B**
Maximum marks for each part of a question are clearly shown
This paper consists of **THREE** printed pages

SECTION A (Compulsory)

QUESTION ONE (30 MARKS)

a) Define a partial differential equation and give an example [2 Marks]

hence find the differential equation arising from $\phi\left(x+y+z, x^2+y^2-z^2\right)=0$ (5 marks)

b) Find the integral curves of the equation (5 marks)

$$\frac{dx}{xy}=\frac{dy}{y^2}=\frac{dz}{xyz-2x^2}$$

c) Show that the equation $\left(x^2z-y^3\right)dx+3xy^2dy+x^3dz=0$ is integrable (3 marks)

d) Show that the wave equation $u_u=a^2u_{xx}$ is variable separable (3 marks)

e) Solve the equation $r+6s+9t=0$ (4 marks)

f) Express the partial differential equation $x\frac{\partial x}{\partial x} y\frac{\partial x}{\partial y} = x^2+y^2$ in polar coordinates and hence solve the equation. (8 marks)

SECTION B (Attempt any TWO questions)

QUESTION TWO (20 MARKS)

a) Find the orthogonal trajectories on the surface $x^2+y^2 2fyz+d=0$ of its intersection with the planes $z=c$ where c is a parameter [10 Marks]

b) Consider a curve which is the intersection of the surfaces $F(x,y,z)=0$ and $G(x,y,z)=0$, prove that $(dx,dy,dz)\alpha\left[\frac{\partial(F,G)}{\partial(y,z)},\frac{\partial(F,G)}{\partial(z,x)},\frac{\partial(F,G)}{\partial(x,y)}\right]$. Hence show that the directional cosine of the tangent at the point $p(x,y,z)$ to the conic $fx^2+gy^2+hz^2=1, x+y+z=1$ are proportional to $(gy-hz, hz-fx-gy)$ (10 marks)

QUESTION THREE (20 MARKS)

a) Solve the following heat distribution equation by the method of separation of variables

$$\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2} -\pi\le x\le\pi,\ t>0$$

$u(0,t)=u(\pi,t)=0 \quad t>0$ and $u(x,0)=\begin{cases} a & -\pi<x<0 \\ -a & 0<x<\pi\end{cases}$ (15 marks)

b) Show that $v=f(y-3x)$, where *f* is arbitrary function is the general solution of the equation

$$\frac{\partial v}{\partial x}+3\frac{dv}{dy}=0$$

Hence find a particular solution which satisfies $v(0,y) = 4 \sin y$.

(5 marks)

QUESTION FOUR (20 MARKS)

a) Using Laplace transform method, solve the partial differential equation $\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=u.$ Subject to the initial conditions $u(x,0)=e^{-5x}$ *and* $u(x,t)$ is bounded for $x>0$ $t>0$. (12 marks)

b) Solve $\left(D_x^2+3D_xD_y+2D_y^2\right)_z=12xy$ (8 marks)

QUESTION FIVE (20 MARKS)

a) Find in its simplest form the integral surface of $z=px+9y+pq$ (whose solution is $z=ax+by+ab$) passing through the curve $x=\tau,\ y=\tau,\ z=\tau^2$ (10 marks)

b) Find the partial differential equation arising from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (5 marks)

c) Solve by direct integration $\frac{\partial^3 z}{\partial x^2\partial y}+18xy^2+\sin(2x-y)=0$ (5 marks)