# THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

**(A Constituent College of Jkuat)**

***Faculty of Applied & Health Sciences***

DEPARTMENT OF MATHEMATICS & PHYSICS

**UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN BUILDING & CIVIL ENGINEERING** (YR III, SEM 1)

## SMA 2471: NUMERICAL ANALYSIS I

SPECIAL/SUPPLEMENTARY EXAMINATION
**SERIES:** FEBRUARY/MARCH 2012
**TIME:** 2 HOURS

**Instructions to Candidates:**
You should have the following for this examination
- Answer booklet

This paper consists of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
This paper consist of **TWO** printed pages

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**QUESTION ONE (30 MARKS)**

a) Show that there exists a root to the quadratic equation $2x^2 - 6x - 3 = 0$ between $x = 3$ and $x = 4$.
Use Newton Raphson method to find the positive root of the equation $2x^2 - 6x - 3 = 0$ correct to 3 significant figures. (Take $x_0 = 3$) [5 Marks]

b) For the table given below, find $\cos x^0$ using the quadratic Newton's forward difference interpolating polynomial. [5 Marks]

| $x^0$ | 30 | 60 | 90 |
|---|---|---|---|
| $\cos x^0$ | 0.866 | 0.500 | 0.000 |

c) Use Simpson's rule to approximate $\int_2^4 \sqrt{(1+x)}\,dx$ with step size $h = 0.5$ [4 Marks]

d) Given the following values of $f(x) = x^2$, find an approximate value of $f'(2.0)$ [5 Marks]

| $x$ | 2.0 | 2.2 | 2.6 |
|---|---|---|---|
| $f(x)$ | 4.00 | 4.840 | 6.76 |

e) An integral related to the complete elliptic integral is defined by

$$K(k) = \int_0^{2\pi} \frac{dx}{\left[1-(\sin k \sin x)^2\right]^{1/2}}$$

From a table of values of these integrals, we find that for various values of $k$ measured in degrees, $K(1) = 1.5709$, $K(4) = 1,5727$ and $K(6) = 1.5751$. Find $K(3.5)$, using a second –degree interpolating polynomial. [5 Marks]

f) Investigate the convergence of $F(x) = \left(\frac{7x-2}{2}\right)^{\frac{1}{3}}$ for the equation $2x^3 - 7x + 2 = 0$ within the closed intervals $[-2,-1]$ and $[0,1]$ [6 Marks]

**QUESTION TWO (20 MARKS)**

a) Show that the truncation error for the trapezoidal rule, over the range of integration $[a,b]$ is $-\frac{1}{12}h^2(b-a)\max f''(\xi), \quad a < \xi < b$

[8Marks]

b) Estimate $\int_0^1 e^{-x^2} dx$ correct to two decimal places using trapezoidal rule [12 Marks]

**QUESTION THREE (20 MARKS)**

a) Apply Gauss – Chebyshev quadrature 3-point formula to evaluate

$$\int_{-1}^{1} \frac{x^4}{\sqrt{(1-x^2)}} dx$$

[5 Marks]

b) Determine $a, b, c$ so that $\int_0^{2h} f(x)dx = af(0) + bf(h) + cf(2h) + E$ is exact [15 Marks]

**QUESTION FOUR (20 MARKS)**

a) Evaluate $\int_0^1 \frac{1}{1+x}\,dx$ using Gause-Legendre 3-point formula [4Marks]

b) Apply the classical fouth order Runge-Kutta method to approximate the solution to the initial value problem

$$\frac{dy}{dt} = \left(\frac{y}{t}\right)^2 + \left(\frac{y}{t}\right), \quad 1 \le t \le 1.2, \quad y(1) = 1, \quad h = 0.1$$

[16Marks]

**QUESTION FIVE (20 MARKS)**

a) Solve by Taylor series the differential equation $xy' = x - y; \quad y(2) = 2$ at $x = 2.1$, correct to 5 decimal places. [13 Marks]

b) Use the Euler method to find an approximation to the initial value problem $\frac{dy}{dx} = y - \frac{2x}{y}, \quad y(0) = 1$ in the range $0 \le x \le 0.2$ with step size $h = 0.1.$ [7 Marks]