THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of JKUAT)
Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF TECHNOLOGY (INFORMATION TECHNOLOGY)

ICS 2211: NUMERICAL LINEAR ALGEBRA

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: MAY/JUNE 2012
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer Booklet*

This paper consists of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are clearly shown
This paper consists of **FOUR** printed pages

Question 1 (Compulsory - 30 Marks)

$$A = \begin{bmatrix} -2 & 1 & -1 & 4 \\ 1 & 2 & 3 & 13 \\ 3 & 0 & 1 & -1 \end{bmatrix}$$

a) (i) Convert the following matrix to row-echelon form (6 marks)
(ii) Solve the following system of equations using your answer in a(i)

$$-2x + y - z = 4$$
$$x + 2y + 3z = 13$$
$$3x + z = -1$$

(3 marks)

b) Given

$$A = \begin{bmatrix} 6 & 0 & 0 & 0 & 0 \\ 9 & -4 & 0 & 0 & 0 \\ -2 & 0 & 11 & 0 & 0 \\ 1 & -1 & 3 & 0 & 0 \\ 0 & 1 & -7 & 4 & 8 \end{bmatrix}$$

find the eigenvalues of B =A^2 (4 marks)

c) (i) Obtain the inverse of the following matrix using the method of co-factors

$$A = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 2 & 2 \\ 5 & 0 & -1 \end{bmatrix}$$

(8 marks)

(ii) Use your answer in b(ii) above to solve the following system of equations

$$\begin{aligned} 3x + y &= 6 \\ -x + 2y + 2z &= -7 \\ 5x - z &= 10 \end{aligned}$$

(3 marks)

b) Determine an LU-Decomposition of the following matrix

$$A = \begin{bmatrix} 3 & 6 & -9 \\ 2 & 5 & -3 \\ -4 & 1 & 10 \end{bmatrix}$$

(6 marks)

Question 2 (20 Marks

a) Use row reduction to compute the determinant of the following matrix

$$A = \begin{bmatrix} 3 & 0 & 6 & -3 \\ 0 & 2 & 3 & 0 \\ -4 & -7 & 2 & 0 \\ 2 & 0 & 1 & 10 \end{bmatrix}$$

(6 marks)

b) Use Cramer's rule to determine the following system of equations:

$$3x_1 - x_2 + 5x_3 = -2$$
$$-4x_1 + x_2 + 7x_3 = 10$$
$$2x_1 + 4x - x_3 =$$

c) Solve the following system of equations using the Gauss-Jordan method

$$2x_1 - x_2 + 2x_3 = 12$$
$$x_1 + 2x + 3x_3 = 11$$
$$2x_1 - 2x_2 - x_3 = 2$$

(6 marks)

Question 3 (20 Marks)

a) Use row reduction to obtain the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & -3 \\ 4 & 1 & 8 \end{bmatrix}$$

(6 marks)

b) Determine the eigenvalues and corresponding eigenvectors for the following matrix:

$$A = \begin{bmatrix} 6 & 16 \\ -1 & -4 \end{bmatrix}$$

(8 marks)

c) Use 3-digit rounded precision and pivoting to solve the system Ax = b given

$$A = \begin{bmatrix} 3.02 & -1.05 & 2.53 \\ 4.33 & 0.56 & -1.78 \\ -0.83 & -0.54 & 1.47 \end{bmatrix} \; and \; B = \begin{bmatrix} -1.61 \\ 7.23 \\ -3.38 \end{bmatrix}$$

(6 marks)

Question 4 (20 Marks)

Given the following system of equations

$$8x_1 + x_2 - x_3 = 8$$
$$2x_1 + x_2 + 9x_3 = 12$$
$$x1 - 7x_2 + 2x_3 = -3$$

(i) Determine the initial solution equations for use by method of iteration (3 marks)

(ii) Determine the Gauss-Sidell iteration equations (3 marks)

$$x_1 = x_2 = x_3 = 0$$

(iii) Solve the system by Gauss-Seidel iterative method starting with

(14 marks)

Question 5 (20 Marks)

a) Explain the terms:
   (i) Pivoting,
   (ii) Ill-conditioned systems as applied to solution of equations (4 marks)

$$A = \begin{bmatrix} -0.002 & 4.000 & 4.000 \\ -2.000 & 2.906 & -5.387 \\ 3.000 & -4.031 & -3.112 \end{bmatrix}, \; B = \begin{bmatrix} 7.988 \\ -4.481 \\ -4.143 \end{bmatrix}$$

b) Consider the sytem Ax = b where

   (i) Using a three-digit chopped arithmetic, solve for x (12 marks)
   (ii) Given that $\underline{x}' = (1.000,\; 1.000, 1.000$ determine $e = x - \underline{x}'$ and $\|e\|$ (3 marks)
   (iii) Is the system ill-conditioned? (Verify your answer) (1 mark)