THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of JKUAT)

(A Centre of Excellence)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE IN BACHELOR OF TECHNOLOGY

ICS 2211: NUMERICAL LINEAR ALGEBRA

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: OCTOBER 2012
TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One (30 marks)

a) Evaluate:

$$\begin{vmatrix} 2 & 5 & -3 & -2 \\ -2 & -3 & 2 & -5 \\ 1 & 3 & -2 & 2 \\ -1 & -6 & 4 & 3 \end{vmatrix}$$

(5 marks)

b) Use Cramer's rule to solve the simultaneous equations:

$$2x + 5y = 4$$
$$-3x - 4y = 1$$

(5 marks)

c) Evaluate $\begin{bmatrix} 6 & 5 \\ -7 & -6 \end{bmatrix}^2$ **(1 mark)**
and hence solve the simultaneous equations.

$$6x + 5y = 7$$
$$7x + 6y = 8$$

(3 marks)

d) Given $A = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 8 & -3 & 0 & 0 & 0 \\ -3 & 0 & 10 & 0 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 1 & 0 & -8 & 3 & 9 \end{bmatrix}$ find the Eigen values. **(4 marks)**

e) Determine an LU-Decomposition of the following matrix.

$$\underset{\sim}{A} = \begin{bmatrix} 4 & 7 & -8 \\ 3 & 6 & -2 \\ -3 & 2 & 11 \end{bmatrix}$$

(6 marks)

f) Find the eigen values and the corresponding eigenvectors of the matrix $\begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}$ **(6 marks)**

SECTION B (Answer any TWO questions from this section)

Question Two (20 marks)

a) Use row reduction to reduce the system given below into an upper triangular matrix and solve it.

$$3x_1 - 4x_2 + x_3 = 5$$
$$-x_1 + 3x_2 + 5x_3 = 16$$
$$4x_1 + 2x_2 - 6x_3 = -8$$

(9 marks)

b) Use the Adjoint matrix method to solve the simultaneous equations.

$$2x_1 + 3x_2 - 4x_3 = 5$$
$$-4x_2 + 2x_3 = -8$$
$$x1 - x_2 + 5x_3 = 9$$

(11 marks)

Question Three (20 marks)

a) Find the eigenvalues and eigenvectors for the matrix:

$$\underset{\sim}{A} = \begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$$

(10 marks)

b) Use Cramer's rule to solve the system of linear equation:

$$2x + 3y - z = 1$$
$$3x + 5y + 2z = 8$$
$$x - 2y - 3z = -1$$

(10 marks)

Question Four (20 marks)

$$\underset{\sim}{A} = \begin{bmatrix} -6 & 4 \\ 6 & 2 \end{bmatrix} \qquad P(A) = -5x^2 + 9x - 8$$

a) Given that , evaluate **(5 marks)**

b) Convert the following matrix into row-echelon form.

$$\begin{bmatrix} -2 & 1 & -1 & 4 \\ 1 & 2 & 3 & 13 \\ 3 & 0 & 1 & -1 \end{bmatrix}$$

(5marks)

c) Solve the following system of equations using your result in 4(b)

$$-2x + y - z = 4$$
$$x + 2y + 3z = 13$$
$$3x + z = -1$$

(3 marks)

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 9 & 5 & 4 \end{bmatrix}$$

d) Use row reduction to find the inverse of the matrix **(7 marks)**

Question Five (20 marks)

a) The solution to the system of equations having the form AX = B can be found by matrix multiplication.

$$X = \begin{bmatrix} 0 & -1 & 1 \\ -2 & 1 & 2 \\ 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$$

i) Find the original system of equations **(8 marks)**

ii) Find the solution of the system **(3 marks)**

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & 2 & 2 \\ 4 & 5 & 7 \end{bmatrix}$$

iii) Find the cofactor matrix of and hence find its inverse **(7 marks)**

b) Define the term pivoting as used in solutions of equations **(2 marks)**