TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE MATHEMATICS & COMPUTER SCIENCE (BMCS)

AMA 4326: FLUID MECHANICS II

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Define the following terms:
- **(i)** Incompressible flow **(1 mark)**
- **(ii)** Equipotential line **(1 marks)**

b) Discuss the flow whose complex potential function is given by $w = z^2$ **(4 marks)**

c) If there is a line source of strength-m at a point Z and a line source of equal strength at the mirror image of z_1 at the line x = 0. Prove that there is no fluid motion across the mirror x = 0 **(6 marks)**

d) Show whether the function $\psi = A(x^2 - y^2)$ represents a possible irrotational flow **(3 marks)**

e) Briefly describe the following standard comformal transformations:
- **(i)** Inversion

(ii) Enlargement **(1 marks)**

$$\frac{U}{U\max} = 1 - \left(\frac{r}{R}\right)^n$$

f) The velocity distribution in a pipe is given by where Umax is the maximum velocity at the centre of the pipe, U is the velocity at a distance r from the centre and R is the pipe radius. Obtain an expression for mean velocity in terms of Umax and n **(5 marks)**

$$\phi = x(2y-1)$$

g) The velocity potential for a two-dimensional flow, is . At the point P(4, 5) determine:

(i) The velocity **(4 marks)**

(ii) The value of the stream function **(4 marks)**

Question Two

$$w = \phi + i\psi$$

a) Prove that is an analytic function **(4 marks)**

$$U = 4ax(x^2 - 3y^2) \qquad V = 4ay(3x^2 - y^2)$$

b) Given that and examine whether these velocity components represent a physically possible 2 dimensional flow. if so whether the flow is rotational or irrotational **(5 marks)**

$$\phi = x^2 - y^2$$

c) For a 2 dimensional flow the velocity function is given by the expression :

(i) Determine velocity components in x and y directions **(2 marks)**

(ii) Show that the velocity components satisfy the conditions of flow continuity and irrorationality, **(4 marks)**

(iii) Determine stream function and the flow rate between the streamlines (2, 0) and (2, 2) **(3 marks)**

(iv) Show that the streamlines and potential lines intersect orthogonally at the point (2, 2) **(2 marks)**

Question Three

a) In a two dimensional flow the velocity components are U = Cy (where C is a constant and V = 0. Find

$$x^2 + y^2 - 2\alpha y = 0 \qquad \alpha$$

the circulation about the circle situated in the flow if is the radius of the circle **(5 marks)**

b) Suppose that a liquid is in the region of the positive side of x – axis with rigid boundary y = 0 and there is a source of strength –m at (0, a) and an equal sink at (0, b). If the pressure on the negative side of the boundary is the same as the pressure at infinity. Show that if the fluid satisfies the non-slip condition on the boundary . Then the resultant pressure on the boundary is given by:

$$P = \frac{1}{2}\rho\int_{-\infty}^{\infty} \frac{4m^2x^2(b^2 - a^2)}{(x^2 + a^2)^2(x^2 + b^2)^2}\,dx$$

(10 marks)

c) Two parallel plates kept 100mm apart have laminar flow of oil between them with a maximum velocity of 1.5m/s. Calculate:

(i) The discharge per metre width **(2 marks)**

(ii) The shear stress at the plates **(3 marks)**

Question Four

a) Discuss the flow due to a uniform line doublet at point O of strength μ per unit length if its axis is along the x-axis. **(7 marks)**

b) Obtain the velocity profile of a plane parallel flow (coutte flow) between two plates moving at a constant velocity U_o but in opposite direction, the distance y is measured from a fixed point 0 and the distance between the plates is 8mm if $\frac{\partial p}{\partial x} \neq 0$, obtain the skin friction on each of the plates. **(7 marks)**

c) Find a relevant stream function for a set of velocity components $u = \frac{-cx}{y}$ and $V = C \ln xy$ to obtain a steady incompressible flow **(6 marks)**

Question Five

a) Discuss the complex velocity potential of an inclined flow to the x-axis at an angle α^o if $\nabla \phi = -\vec{q}$ **(8 marks)**

b) Fluid is in laminar motion between two parallel plates under the action of motion of one of the plates and also under the presence of a pressure gradient in such a way that the net forward discharge across any section is zero:
   (i) Find out the point where minimum velocity occurs and its magnitude **(7 marks)**
   (ii) Draw the velocity distribution graph across a section of the parallel plates **(2 marks)**

c) Write the complete Navier stokes equation of conservation of momentum and name all the terms in the equation **(3 marks)**